Dynamic Collective Argumentation: Constructing the Revision and Contraction Operators

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Abstract

Collective argumentation has always focused on obtaining rational collective argumentative decisions. One approach that has been extensively studied in the literature is the aggregation of individual extensions of an argumentation framework. However, previous studies have only examined aggregation processes in static terms, focusing on preserving semantic properties at a given time. In contrast, this paper investigates whether decisions remain rational when the preservation process is dynamic, meaning that it can incorporate new information. To address the dynamic nature of collective argumentation, we introduce the revision and contraction operators. These operators reflect the idea that when an individual or a group learns something new by accepting or rejecting an argument, they have to update their collective decision accordingly. Our study examines whether the order of revising individual opinions and aggregating them affects the final outcome, i.e., whether aggregation and revision commute.

Keywords: argumentation theory, social choice theory, belief revision

1. Introduction

Group decision-making processes involve a complex interplay of various factors, including individual perspectives, beliefs, and values. As groups navigate these intricate dynamics, they are constantly exposed to new information that can challenge their assumptions and alter their opinions. This new information can come in various forms, such as compelling arguments, emerging evidence, or changing circumstances. One critical aspect of this new information is its acceptability - that is, whether it is perceived as credible, reliable, and relevant by group members. When new information is deemed acceptable, it has the potential to persuade individuals that their current position is wrong or incomplete, leading them to adjust their opinions accordingly. This process of opinion revision is crucial for ensuring that group decisions reflect the most accurate and up-to-date information available.

Upon receiving new information, the group can integrate it in one of two ways. The first approach involves individual members revising their opinions based on the new information, followed by the group aggregating these post-revision opinions. Alternatively, the group may first aggregate its members' opinions prior to revision, and then revise the collective decisions based on the new information. To illustrate the impact of new information on group decision-making, consider the following scenario:

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Example 1. Three urban planners, designated as Planner 1, Planner 2, and Planner 3, are engaged in a discussion to enhance the urban environment and transportation systems of a major city. They are considering three distinct development strategies:

- Argument A: Expansion of Public Transit Systems.
- Argument B: Development of Bicycle Lanes and Pedestrian Paths.
- Argument C: Implementation of Congestion Pricing.

Their goal is to reach a collective decision by aggregating individual stances on these arguments. The initial support distribution is as follows:

- Planner 1 supports A and B.
- Planner 2 initially supports B and C.
- Planner 3 supports A and C.

Now, suppose the group of planners learns new information – data demonstrating the effectiveness of public transit systems in other cities – prompting a revision of their stances. If the group first aggregates planners' pre-revision stances and then revises the resulting collective stances based on the new information, the outcome will be $\{A, B, C\}$. This is because A is already included in the collective stances. Alternatively, if the group members first revise their individual stances based on the newly learnt information, and the group then aggregates their post-revision stances, the result will be $\{A, B\}$. Planner 2, influenced by the data, now endorses A and withdraws support for C, aiming for strategic simplification. Consequently, C is no longer supported by a majority of planners. Thus, the two approaches, revision followed by aggregation and aggregation followed by revision, led to different outcomes. △

This example demonstrates how different approaches to integrating new information can lead to distinct outcomes in decision-making processes. In this paper, we consider a group of agents, each with their own sets of supported arguments in an argumentation framework [34]. Abstract argumentation is a mathematically elegant formalism that allows us to focus on fundamental principles. An abstract argumentation framework is simply a set of arguments with a binary attack-relation defined on this set. When the agents learn a new argument, they need to update their individual sets accordingly. We investigate whether the new collective decisions are the old ones revised in light of this information. If the order of revising individual opinions and aggregating them does not affect the final outcome, i.e., whenever all individuals revise their individual extensions in light of some information (a learnt argument), then the new aggregate extension are the old ones revised in light of this information, then we say that the aggregation rule is dynamically rational.

We propose two types of operators, the revision and contraction operators. These operators are used to modify an extension in response to new information, either by adding or removing an argument. The revision operator expresses the idea that when individuals learn something new by accepting an argument, they need to add an argument to an extension. The contraction operator expresses that when individuals learn that an argument is unacceptable, they need to remove an argument from an extension. These operators reflect the idea that when an individual or a group learns something new by accepting or rejecting an argument, they have to update their collective decision accordingly. This idea is adapted from the belief revision theory in the AGM framework [1].

We examine the preservation of semantic properties of extensions of argumentation frameworks. We indicate that, when considering a set of reasonable revision operators, dynamic rationality and some reasonable axioms cannot be satisfied simultaneously, at least when the semantic property we hope to aggregate is completeness or admissibility, two fundamental components of most semantics of argumentation frameworks. For instance, we demonstrate that any aggregation rule satisfying the properties of being grounded, independent, and neutral, while also being dynamically rational with respect to a completeness-preserving operator and preserving completeness, must be dictatorial. This implies that there will always be a single agent who can determine the outcome. Similarly, if an aggregation rule satisfies several fundamental axiomatic properties, and dynamically rational with respect to a contraction admissibility-preserving operator, while guaranteeing admissibility for certain argumentation frameworks, it must also be dictatorial.

However, if we eliminate argumentation frameworks that are not desirable, dynamically rational aggregation becomes possible. For example, we may assume that argumentation frameworks containing cycles are irrational. In this case, we can study dynamic rationality in the context of acyclic argumentation frameworks. For acyclic argumentation frameworks, the acceptance status of arguments is unambiguous. If the argumentation framework under consideration satisfies such properties, then dynamic rationality can be preserved. For example, if an aggregation rule is dynamically rational with respect to specific revision or contraction operators, then such a rule guarantees the most demanding semantic properties for acyclic argumentation frameworks.

The paper is structured as follows: Section 2 reviews the fundamental concepts of the abstract argumentation model, which was originally introduced by Dung [34]. Section 3 describes a model for aggregating individual argumentative stances and the concept of dynamic rationality. Section 4 and Section 5 explore how adding and removing an argument can affect the outcome and whether aggregation and revision commute. Section 6 studies dynamic rationality with restricted argumentation frameworks. Section 7 explores dynamic rationality under the assumption that individual extensions are complete. Section 8 provides some related work. Finally, Section 9 concludes the paper.

2. Abstract argumentation

This section reviews the fundamental concepts of the abstract argumentation model, which was originally introduced by Dung [34]. An argumentation framework consists of a finite set of arguments, denoted by Arg, and an irreflexive binary relation on Arg called $\rightharpoonup$. For $\Delta \subseteq Arg$ and $B \in Arg$, we write $\Delta \rightharpoonup B$ (namely, Δ attacks B) in case $A \rightharpoonup B$ for at least one argument $A \in \Delta$. For $\Delta \subseteq Arg$ and $C \in Arg$, if Δ attacks all arguments $B \in Arg$ such that $B \rightharpoonup C$, we say that Δ defends C. Finally, 2^{Arg} denotes the powerset of Arg.

An argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$ presents the question of deciding which subset of arguments $\Delta \subseteq Arg$ should be accepted as an *extension* of AF. Various criteria have been suggested for selecting an extension. Although Dung has established multiple semantics, including complete, grounded, preferred, and stable semantics [34], it should be noted that extensions of semantics are expected to meet fundamental criteria, such as conflict-freeness, self-defense, and admissibility.

Definition 1. *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework and let $\Delta \subseteq Arg$ be a set of arguments. We adopt the following terminology:*

- *Δ is called conflict-free if there are no arguments $A, B \in \Delta$ such that $A \rightharpoonup B$.*
- *Δ is called self-defending if $\Delta \subseteq \{C \mid \Delta$ defends $C\}$.*
- *Δ is called admissible if it is both conflict-free and self-defending.*
- *Δ is called reinstating if $\{C \mid \Delta$ defends $C\} \subseteq \Delta$.*
- *Δ is a complete extension of AF if it is admissible and contains all the arguments it defends.*
- *Δ is a preferred extension of AF if it is $\subseteq$-maximal amongst the admissible extensions.*
- *Δ is a grounded extension of AF if it is $\subseteq$-minimal amongst the complete extensions.*
- *Δ is a stable extension of AF if it is conflict-free and attacks all the arguments in $Arg \backslash \Delta$.*

For a given argumentation framework, it is possible that the set of stable extensions is empty, while there always exists at least one extension under complete, preferred, and grounded semantics. In addition, there is always a single grounded extension, although it is worth noting that this extension can sometimes be empty. Before concluding this section, it is important to note that in the argumentation frameworks we discuss, attack relations are considered to be irreflexive. While this assumption of irreflexivity is not essential to the outcomes of our study, it does help to clarify the presentation, prevent counterintuitive scenarios during the revision and contraction processes, and is generally a natural fit for most practical uses. Additionally, the irreflexivity assumption brings other benefits, such as reducing complexity in decision-making processes and eliminating self-attacking arguments [6], which are typically not found in all settings, especially in classical logic-based frameworks where they do not occur at all (cf. [[7], Theorem 4.13]).

3. Dynamic rationality

Assume an argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$ and a finite set of agents $N = \{1, \ldots, n\}$. Each agent $i \in N$ provides us with an extension $\Delta_i \subseteq Arg$ that represents their personal views about what constitutes an acceptable set of arguments in the context of AF. Therefore, we have a *profile* $\boldsymbol{\Delta} = (\Delta_1, \ldots, \Delta_n)$, which is a vector of extensions, with one extension for each agent. An *aggregation rule* is a function $F : (2^{Arg})^n \to 2^{Arg}$ that maps a profile of extensions to a single extension.

One example of an aggregation rule is the *majority rule*, which includes an argument $A \in Arg$ in the output extension if and only if it is present in a strict majority of the individual extensions. Given a profile of extensions $\boldsymbol{\Delta}$ and an argument $A \in Arg$, the set of agents who accept A under the profile $\boldsymbol{\Delta}$ is denoted by $N_A^{\boldsymbol{\Delta}}$, which is defined as $N_A^{\boldsymbol{\Delta}} = \{i \in N \mid A \in \Delta_i\}$.

Definition 2 (Quota rules)**.** *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework, let N be a set of n agents, and let $q \in \{1, \ldots, n\}$. The* **quota rule** *F_q with quota q is defined as the aggregation rule mapping any given profile $\boldsymbol{\Delta} = (\Delta_1, \ldots, \Delta_n) \in (2^{Arg})^n$ of extensions to the extension including exactly those arguments accepted by at least q agents:*

$$F_q(\boldsymbol{\Delta}) \quad = \quad \{A \in Arg \mid \#\{N_A^{\boldsymbol{\Delta}}\} \geqslant q\}$$

Such quota rules are sometimes more precisely referred to as *uniform* quota rules, to stress the fact that the acceptance of each argument is subject to *the same* quota q.
We now introduce another family of aggregation rules, namely the family of oligarchic rules.

Definition 3 (Oligarchic rules)**.** *Let $\mathfrak{C} \in 2^N \setminus \{\emptyset\}$ be a nonempty coalition of agents. The* **oligarchic rule** *$F_{\mathfrak{C}}$ accepts all those arguments that are accepted by all members of $\mathfrak{C}$:*

$$F_{\mathfrak{C}}(\mathbf{\Delta}) = \{A \in Arg \mid \mathfrak{C} \subseteq N_A^{\mathbf{\Delta}}\}$$

Thus, any member of the oligarchy $\mathfrak{C}$ can veto the acceptance of any given argument. The *dictatorship* of dictator $i \in N$ is the oligarchic rule $F_{\mathfrak{C}}$ with $\mathfrak{C} = \{i\}$. Thus, under a dictatorship, to compute the outcome, we simply copy the extension of the dictator. Oligarchic rules, and dictatorships in particular, are unattractive rules as they unfairly exclude everyone not in $\mathfrak{C}$ from participating in the decision process.

We introduce several desirable properties of aggregation rules. Such properties are called axioms in social choice theory. All of them are adapted from judgment aggregation [37], and have also been discussed in other parts of social choice theory, such as preference aggregation [2] and graph aggregation [36].

Definition 4. *An aggregation rule F is* **independent** *if and only if, for any two profiles $\mathbf{\Delta}, \mathbf{\Delta'} \in (2^{Arg})^n$ and any argument $A \in Arg$, it is the case that $N_A^{\mathbf{\Delta}} = N_A^{\mathbf{\Delta}'}$ implies $A \in F(\mathbf{\Delta}) \Leftrightarrow A \in F(\mathbf{\Delta'})$.*

Definition 5. *An aggregation rule F is* **neutral** *if and only if, for any profile $\mathbf{\Delta} \in (2^{Arg})^n$ and any two arguments $A, A' \in Arg$, it is the case that $N_A^{\mathbf{\Delta}} = N_{A'}^{\mathbf{\Delta}}$ implies $A \in F(\mathbf{\Delta}) \Leftrightarrow A' \in F(\mathbf{\Delta})$.*

Definition 6. *An aggregation rule F is* **grounded** *if and only if, for all profiles $\mathbf{\Delta} = (\Delta_1, \ldots, \Delta_n) \in (2^{Arg})^n$, it is the case that $F(\mathbf{\Delta}) \subseteq \Delta_1 \cup \cdots \cup \Delta_n$.*

Definition 7. *An aggregation rule F is* **unanimous** *if and only if, for all profiles $\mathbf{\Delta} = (\Delta_1, \ldots, \Delta_n) \in (2^{Arg})^n$, it is the case that $F(\mathbf{\Delta}) \supseteq \Delta_1 \cap \cdots \cap \Delta_n$.*

Definition 8. *An aggregation rule F is* **monotonic** *if and only if, for any two profiles $\mathbf{\Delta}, \mathbf{\Delta'} \in (2^{Arg})^n$ and any argument $A \in Arg$, it is the case that $N_A^{\mathbf{\Delta}} \subset N_A^{\mathbf{\Delta}'}$ implies $A \in F(\mathbf{\Delta}) \Rightarrow A \in F(\mathbf{\Delta'})$.*

The concept of *independence* implies that the acceptance of an argument depends solely on the individuals who support it. *Neutrality* requires that all arguments be treated equally. A *grounded* aggregation rule accepts only those arguments that have support from at least one individual, while a *unanimous* aggregation rule accepts all arguments that have support from all individuals. An aggregation rule is *monotonic* if adding more supports to an accepted argument does not cause it to be rejected.

Definition 9 (Guarantee)**.** *Let AF be an argumentation framework. Given a semantic property $\delta \subseteq 2^{Arg}$ and an aggregation rule $F : (2^{Arg})^n \to 2^{Arg}$ for n agents, we say that F* **guarantees** *δ if for all profiles $\mathbf{\Delta} = (\Delta_1, \ldots, \Delta_n)$, we have $F(\mathbf{\Delta}) \in \delta$.*

An aggregation rule guarantees a semantic property δ only when the output satisfies δ, regardless of whether the individual extensions meet δ.

Definition 10 (Preservation)**.** *Let AF be an argumentation framework. Given a semantic property $\delta \subseteq 2^{Arg}$ and an aggregation rule $F : (2^{Arg})^n \to 2^{Arg}$ for n agents, we say that F* **preserves** *δ if for all profiles $\mathbf{\Delta} = (\Delta_1, \ldots, \Delta_n) \in \delta^n$, we have $F(\mathbf{\Delta}) \in \delta$.*

An aggregation rule preserves a semantic property when every individual extension satisfies δ, the output satisfies δ.

Given an extension $\Delta \subseteq Arg$ and an argument $A \in Arg$, an operator $\mid$ is a function that assigns a new extension $\Delta \mid A$ to (Δ, A). The operator expresses the idea that when an individual or a group learns something new by accepting or rejecting an argument, they have to update their collective decision accordingly. This idea is adapted from the belief revision theory in the AGM framework [1].

Definition 11. *Given a set of arguments $\Delta \subseteq Arg$, an argument $A \in Arg$, and a semantic property δ, an operator $\mid$ is δ-**preserving** if whenever Δ satisfies δ, $\Delta \mid A$ satisfies δ for any extension $\Delta \subseteq Arg$ and any argument $A \in Arg$.*

Hence, an operator is considered to be δ-preserving if it can preserve specific properties of δ during the process of revision. For instance, revising a set of arguments that is conflict-free will result in another conflict-free set. In this case, the property that we want to preserve is conflict-freeness. Other properties that can be preserved include self-defense, admissibility, completeness, preferredness, and stability. From the perspective of the AGM framework, the requirement of δ-preservation may be considered too demanding for certain properties. Nevertheless, it is a natural condition given that semantic properties hold a central position in abstract argumentation theory.

The requirement of δ-preservation for operators is novel and arguably more demanding compared to the requirements for AGM belief revision operators. In addition to this requirement, there are two more notable differences between AGM belief revision operators and operators in this paper. Firstly, while AGM belief revision operators are typically defined for beliefs over an entire logic of propositions, in this case, a revision operator is specifically defined for extensions of argumentation frameworks in the form $\Delta \subseteq Arg$. Additionally, belief sets in AGM belief revision theory are deductively closed, acondition that sets of arguments in this paper do not fulfill.

In addition to these classical properties, we also introduce further preservation conditions for operators.

Definition 12. *Let $\mid$ be an operator. An argument $A \in Arg$ is called a **fixed defender** of an argument $C \in Arg$ with respect to an attacker $B \in Arg$ and $\mid$ if: (i) $B \rightharpoonup C$ and $A \rightharpoonup B$, and (ii) A is in the set $\Delta \mid C$, where $\Delta \subseteq Arg$.*

This definition implies that a fixed defender A is an argument that not only attacks the attacker B of C but is also included in the set resulting from the application of the operator $\mid$ to C within the context of Δ.

Definition 13. *An operator $\mid$ is called a **defender-fixed** operator if, given a set of arguments $\Delta \subseteq Arg$ and an argument $C \in Arg$, for every attacker $B \in Arg$ of C, if B has at least one attacker, then there exists a fixed defender A of C with respect to B and $\mid$ such that $A \in \Delta \mid C$.*

Thus, for any set of arguments $\Delta \subseteq Arg$, when an argument C is attacked by an argument B, the defender-fixed operator ensures that there is a fixed defender A for C that attacks B and is included in the new extension $\Delta \mid C$, i.e, $A \in \Delta \mid C$.

Definition 14. *Given a set of arguments $\Delta \subseteq Arg$ and an argument $A \in Arg$, an operator $\mid$ is called a **two-defender** operator if given a set of arguments $\Delta \subseteq Arg$ and an argument $C \in Arg$, for every C's attacker $B \in Arg$, if B has at least two attackers, then there is a pair of B's attackers $A_1, A_2 \in Arg$ such that at least one of A_1 and A_2 is included in $\Delta \mid C$.*

By requiring that at least one of the attackers of B is included in the new extension $\Delta \mid C$, the two-defender operator ensures that the extension can defend against attacks from different angles.

Definition 15. *Let $\mid$ be an operator. An argument $B \in Arg$ is called a* **fixed attacker** *of an argument $C \in Arg$ if the following conditions are met: (i) $B \rightharpoonup C$, and (ii) for any argument $A \in Arg$, if $A \rightharpoonup B$, then $A \notin \Delta \mid C$ where $\Delta \subseteq Arg$.*

In simpler terms, a fixed attacker B of an argument C is an argument that attacks C and every B's attacker is excluded from the set resulting from the application of the operator $\mid$ to C within the context of Δ.

Definition 16. *An operator $\mid$ is called an* **attacker-fixed** *operator if, given a set of arguments $\Delta \subseteq Arg$ and an argument $C \in Arg$, if C has at least one attacker, then there is a C's fixed attacker B such that for every B's attacker A, $A \notin \Delta \mid C$.*

Thus, if B is the argument that attacks C and remains "fixed", then for any argument A that attacks B, A is not in the set. In other words, all B's attackers are excluded in $\Delta \mid C$.

Definition 17. *An operator $\mid$ is called a* **two-attacker** *operator if given a set of arguments $\Delta \subseteq Arg$ and an argument $C \notin \Delta$, if C has at least two attackers, then there exists a pair of attackers, B_1 and B_2, for C, such that at least one of B_1 and B_2 has all of their attackers excluded in $\Delta \mid C$.*

Thus, the two-attacker operator requires the existence of a pair of attackers for an argument in order to exclude the entire set of attackers of that argument from the new extension.

Definition 18 (Dynamic rationality)**.** *Given a profile $\mathbf{\Delta}$, a learnt argument $B \in Arg$ and an operator $\mid$, we say that an aggregation rule is* **dynamically rational** *if $F(\Delta_1 \mid B, \cdots, \Delta_n \mid B) = F(\Delta_1, \cdots, \Delta_n) \mid B$.*

Thus, when each individual set of arguments is revised in light of a new argument that is learnt, the collective set should be the same as the old ones revised in light of this information. In other words, it does not matter whether the individuals revise their arguments first and then combine them, or combine them first and then revise them, dynamic rationality assumes that the result should be the same.

Intuitively, dynamic rationality means that the group can update its collective opinion in a coherent and reasonable way, without being influenced by the order of the revision and aggregation processes. For example, suppose a group of agents have different opinions on what constitutes an acceptable set of arguments in the context of a given AF, and they use a certain rule to aggregate their opinions into a collective one. Now, suppose they learn a new argument A that affects their individual opinions. If the aggregation rule is dynamically rational, then the new collective opinion should be the same regardless of whether they first revise their individual opinions based on A and then aggregate them, or first aggregate their individual opinions and then revise the collective one based on B. This way, the group can avoid any potential inconsistencies or paradoxes that may arise from different ways of updating their collective opinion.

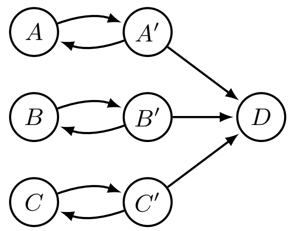

Figure 1: An argumentation framework with seven arguments

	Pre-Learn. A			Post-Learn. A		
	A	B	C	A	B	C
Individual 1	✓	✓	×	✓	✓	×
Individual 2	×	✓	✓	✓	✓	×
Individual 3	✓	×	✓	✓	×	✓
Majority rule	✓	✓	✓	✓	✓	×

Table 1: Table used in Example 2

4. Revision

In this section, we introduce the revision operator. The revision operator expresses the idea that when individuals learn something new by accepting an argument, they need to adjust their collective decision in response.

Definition 19. *Given an extension $\Delta \subseteq Arg$ and an argument $A \in Arg$, a* **revision operator** $\dashv$ *is a function that assigns a new extension $\Delta \dashv A$ to (Δ, A). We call a revision operator* regular *if it satisfies the following two minimal conditions:*

- (i) *successful, i.e., $A \in \Delta \dashv A$ for any pair (Δ, A);*
- (i) *conservative, i.e., $\Delta = \Delta \dashv A$ iff $A \in \Delta$.*

Thus, a revision operator is successful if the learnt argument is included in the post-revision extension; a revision operator is conservative if the learnt argument is already included, then nothing changes.

In this following, we present an example that examines whether the aggregation results of the individual extensions of the agents after they learn a new argument is the same as the aggregation results before the revision.

Example 2. Take into consideration an argumentation framework $AF = \langle \{A, B, C, A', B', C', D\}, \{A \rightharpoonup A', A' \rightharpoonup A, B \rightharpoonup B', B' \rightharpoonup B, C \rightharpoonup C', C' \rightharpoonup C, A' \rightharpoonup D, B' \rightharpoonup D, C' \rightharpoonup D\}\rangle$, as depicted in Figure 1.

Assume there are three agents, each endorsing distinct complete extensions, namely $\{A, B\}$, $\{B, C\}$, and $\{A, C\}$ for the first, second, and third agents, respectively, and they would like to preserve completeness. According to the majority rule, the outcome of the aggregation is $\{A, B, C\}$, which is not a complete extension.

Assume these agents learnt the acceptability of A, in other words, they have learnt that A is acceptable. Consequently, the second agent modifies his extension to $\{A, B\}$ and rejects C. This is because D is now defended by $\{A, B, C\}$, and accepting A, B, and C without accepting D would violate the semantic property of completeness.[1] In the meantime, the individual extensions of the first and third agents remain unchanged as they have previously accepted A. Consequently, the outcome following the majority rule becomes $\{A, B\}$. This scenario is represented in Table 1.

However, if the agents first aggregate their pre-revision extensions, the resulting majority extension will be $\{A, B, C\}$. Post the revision in response to the acceptance of A, the new extension remains as $\{A, B, C\}$, differing from the pre-revision extension. △

In Example 2, it is evident that dynamic rationality is not upheld under the assumption that the revision operator is successful and conservative, and the aggregation process is completeness-preservation.

By Definition 11, a revision operator is conflict-freeness-preserving if whenever Δ is conflict-free, $\Delta \dashv A$ is conflict-free for any extension $\Delta \subseteq Arg$ and any argument $A \in Arg$. Thus, the concept of a conflict-freeness-preserving revision operator is introduced to ensure that the revised extension remains conflict-free. Since conflict-freeness is a basic property of argument extensions, this type of operator is instrumental in ensuring that the fundamental characteristics of argumentation semantics are preserved during revision.

Given a regular conflict-freeness-preserving revision operator, a set of arguments Δ, and an argument A, if A is included in Δ, then nothing changes. If A is not included in Δ, then A will be included in $\Delta \dashv A$. In the meantime, every argument that conflicts with A will be excluded from $\Delta \dashv A$.

Theorem 1. *For any aggregation rule F, if F is grounded and dynamically rational with respect to a regular conflict-freeness-preserving revision operator, then F guarantees conflict-freeness.*

Proof. Let F be an aggregation rule that is grounded. Suppose we have a profile $\boldsymbol{\Delta} = (\Delta_1, \cdots, \Delta_n)$, $A \in F(\boldsymbol{\Delta})$, and $A' \in Arg$ such that $A \rightharpoonup A'$ or $A' \rightharpoonup A$. We need to show that $A' \notin F(\boldsymbol{\Delta})$. By dynamic rationality, $F(\Delta_1 \dashv A, \cdots, \Delta_n \dashv A) = F(\Delta_1, \cdots, \Delta_n) \dashv A$. In this equation, the right side equals $F(\Delta_1, \cdots, \Delta_n)$ by conservativeness and the fact that $A \in F(\Delta_1, \cdots, \Delta_n)$.

Now, let us turn to consider the left side, if Δ_i includes A', then $A' \notin \Delta_i \dashv A$ by the conflict-freeness revision operator; if Δ_i does not contain A', then $A' \notin \Delta_i \dashv A$ by the conservativeness of the revision operator. Thus, $A' \notin \Delta_i \dashv A$ for all $i \in N$, it follows that $A' \notin F(\Delta_1 \dashv A, \ldots, \Delta_n \dashv A)$ by the groundedness of F. Therefore, $A' \notin F(\boldsymbol{\Delta})$. This completes the proof. □

Thus, a grounded aggregation rule always produces conflict-free outcomes when such a rule is dynamically rational with respect to a regular conflict-freeness-preserving revision operator. If we

[1] After learning that A is acceptable, the second agent needs to revise his extension. In the meantime, we observe that there are more than one possible ways of revision, removing C is one way, and accepting D without removing C is another way, these two ways have no clear advantage over each other, and for the sake of illustrating the concept of dynamic rationality, we assume here that the second agent adopts the former way.

consider the guarantee of conflict-freeness only, Chen and Endriss [17] have shown that the majority rule preserves conflict-freeness, as does every quota rule with an even higher quota.

Theorem 2 (Chen and Endriss, 2018)**.** *Let AF be any argumentation framework with at least one attack between two arguments that do not attack themselves. Then a quota rule F_q for n agents preserves conflict-freeness for AF if and only if $q > \frac{n}{2}$.*

Theorem 2 focuses on whether conflict-freeness is preserved when individual extensions are aggregated into collective ones at a single point in time, which is called *static rationality* in this paper.

According to Definition 13, a revision operator $\dashv$ is referred to as a defender-fixed revision operator when, for a given set of arguments $\Delta \subseteq Arg$ and an argument $C \in Arg$, for every attacker $B \in Arg$ of C, there exists a fixed defender A for C that attacks B and $A \in \Delta \dashv C$. In this case, a defender-fixed argument refers to an argument that attacks B and remains fixed under the operation $\dashv$ when considering an attacker B of argument C. In the following proposition, we assume that the revision operator is a defender-fixed operator.

Proposition 3. *For any aggregation rule F, if F is grounded, unanimous, and dynamically rational with respect to a defender-fixed regular revision operator for arguments, then F preserves self-defense.*

Proof. Let F be the specified aggregation rule, and let $\dashv$ be a defender-fixed regular revision operator. Consider a profile $\boldsymbol{\Delta} = (\Delta_1, \cdots, \Delta_n)$ with Δ_i is self-defending for all $i \in N$, and let $\Delta = F(\boldsymbol{\Delta})$. Our goal is to show that Δ is self-defending, which means that for every argument $C \in \Delta$ such that $B \rightharpoonup C$, there is an argument $A \in \Delta$ such that $A \rightharpoonup B$ holds.

Now we take an argument $C \in \Delta$, and assume $B \rightharpoonup C$. By dynamic rationality of F, we have $F(\Delta_1 \dashv C, \cdots, \Delta_n \dashv C) = F(\Delta_1, \cdots, \Delta_n) \dashv C$. By conservativeness, we know that $F(\Delta_1, \cdots, \Delta_n) \dashv C = F(\Delta_1, \cdots, \Delta_n)$, so the right side of the equation equals $F(\Delta_1, \cdots, \Delta_n)$.

Assume that B has at least one attacker, and assume that A is the defender-fixed argument of C with respect to attacker B, i.e., $A \in \Delta_i \dashv C$ for all $i \in N$. Therefore, $A \in F(\Delta_1 \dashv C, \cdots, \Delta_n \dashv C)$ by the assumption that F is unanimous. Hence, we conclude that $A \in F(\Delta_1, \cdots, \Delta_n)$.

If B has no attacker, i.e., C has no defender w.r.t. B. Then, by the assumption that all individual extensions are self-defending, C is excluded from every individual extension. Hence, under the assumption that F is grounded, we have $C \notin F(\Delta_1, \cdots, \Delta_n)$. This concludes the proof. □

According to Definition 14, a revision operator $\dashv$ is called a two-defender revision operator when presented with a set of arguments $\Delta \subseteq Arg$ and an argument $C \in Arg$. For each attacker $B \in Arg$ of C, there exists a pair of attackers $A_1, A_2 \in Arg$ of B such that at least one of A_1 and A_2 is included in $\Delta \dashv C$. In the following proposition, we assume that the revision operator is a two-defender operator.

Proposition 4. *Given an argumentation framework AF, if the majority rule F is dynamically rational with respect to a two-defender regular revision operator for AF and the number of agents is odd, then F preserves self-defense for AF.*

Proof. Let F be the majority rule and let $\dashv$ be a two-defender regular revision operator. Let $\boldsymbol{\Delta} = (\Delta_1, \cdots, \Delta_n)$ be a profile with Δ_i is self-defending for all $i \in N$, and $\Delta = F(\boldsymbol{\Delta})$ be its outcome. We want to prove that Δ is self-defending, meaning that for any argument $C \in \Delta$ that is attacked by $B \in Arg$, there exists an argument $A \in \Delta$ that attacks B, i.e., C is defended by Δ.

Now we pick an argument $C \in \Delta$ and an attacker B of C[2], and assume that C is attacked by B, we are going to show that B is attacked by some argument $A \in \Delta$. By the assumption that F is dynamically rational, we have $F(\Delta_1 \dashv C, \cdots, \Delta_n \dashv C) = F(\Delta_1, \cdots, \Delta_n) \dashv C$. Also, by conservativeness of the revision operator, we have $F(\Delta_1, \cdots, \Delta_n) \dashv C = F(\Delta_1, \cdots, \Delta_n)$.

Assume B has at least two attackers, and assume that A_1, A_2 are the two-defender arguments of C with respect to attacker B, i.e., for all $i \in N$, either A_1 or A_2 belongs to $\Delta_i \dashv C$. Then, by the assumption of the number of agents is odd and applying the majority rule, either A_1 or A_2 belongs to $F(\Delta_1 \dashv C, \cdots, \Delta_n \dashv C)$. Therefore, either A_1 or A_2 belongs to $F(\mathbf{\Delta})$ and we are done.

Assume B has only one attacker, we denote it by A. Given that C belongs to $F(\mathbf{\Delta})$, C is accepted by a majority of agents. Furthermore, C has only one defender, which is A. Therefore, every agent who accepts C must also accept A, implying that A is also accepted by a majority of agents, that is, $A \in F(\mathbf{\Delta})$.

If B has no attacker, i.e., C has no defender w.r.t. B. Then, by the assumption that all individual extensions are self-defending, C is excluded from every individual extension. Hence, C is not accepted by a majority of agents, i.e., $C \notin F(\Delta_1, \cdots, \Delta_n)$. This concludes the proof. □

Given a set of arguments $\Delta \subseteq Arg$ and an argument $A \in Arg$, by Definition 11, a revision operator is completeness-preserving if whenever Δ is complete, $\Delta \dashv A$ is complete for any extension $\Delta \subseteq Arg$ and any argument $A \in Arg$.

Theorem 5. *No uniform quota rule F_q with $q < n$ is dynamically rational with respect to any regular revision completeness-preserving operator for some argumentation frameworks.*

Proof. Let us consider the argumentation framework $AF = \langle \{A, B, C, A', B', C', D\}, \{A \rightharpoonup A', A' \rightharpoonup A, B \rightharpoonup B', B' \rightharpoonup B, C \rightharpoonup C', C' \rightharpoonup C, A' \rightharpoonup D, B' \rightharpoonup D, C' \rightharpoonup D\}\rangle$ as depicted in Figure 1. Next, consider the set of arguments $\{A, B, C\}$. Since D is defended by $\{A, B, C\}$ but not included in the set, $\{A, B, C\}$ is not complete. However, every proper subset of $\{A, B, C\}$ is complete.

Now, let F_q be a uniform quota rule with $q < n$. Assume for the sake of contradiction that F is dynamically rational, and let the revision operator be completeness-preserving. Then, by Theorem 1 and the fact that every quota rule is grounded, F guarantees conflict-freeness, which implies that $q > \frac{n}{2}$, this also holds by Theorem 2.

For each $Y \in \{A, B, C\}$, fix an extension $\Delta_{\neg Y}$ such that $\{A, B, C\} \backslash Y \subseteq \Delta_{\neg Y}$. Choose A from $\{A, B, C\}$ and note that since $\Delta_{\neg A} \dashv A$ cannot contain all $Y \in \{A, B, C\}$ (as the revision operator preserves completeness) but contains A (as the revision operator is successful), there exists some argument $X \in \{A, B, C\} \backslash \{A\}$ such that $X \notin \Delta_{\neg A} \dashv A$. Without loss of generality, assume that $X = B$, i.e., $B \notin \Delta_{\neg A} \dashv A$. Since $| \{A, B, C\} | \geqslant 3$, we can pick the third argument in $\{A, B, C\} \backslash \{A, B\}$, i.e., pick C.

Consider a profile $\mathbf{\Delta} = (\Delta_1, \cdots, \Delta_n)$ in which $n - q$ individuals hold $\Delta_{\neg A}$, $n - q$ individuals hold $\Delta_{\neg B}$, and all remaining individuals hold $\Delta_{\neg C}$. Since A and B are each accepted by q individuals, $A, B \in F(\Delta_1, \cdots, \Delta_n)$.

Now consider the revised profile $F(\Delta_1 \dashv A, \cdots, \Delta_n \dashv A)$. Since $B \notin \Delta_{\neg A} \dashv A$, and based on the regularity assumption that $\Delta_{\neg B} \dashv A = \Delta_{\neg B}$ and $\Delta_{\neg C} \dashv A = \Delta_{\neg C}$, only those individuals who previously

[2]If C does not have any attacker, then C is defended by Δ trivially.

held $\Delta_{\neg C}$ would accept B in the revised profile. This means that B would only be accepted by $n - 2 \cdot (n - q) = 2 \cdot q - n$ individuals, which is less than q. Therefore, B would not be included in $F(\Delta_1 \dashv A, \cdots, \Delta_n \dashv A)$.

Now, $F(\Delta_1, \cdots, \Delta_n)$ is equal to $F(\Delta_1, \cdots, \Delta_n) \dashv A$ because it contains A and the revision operator is regular. However, it differs from $F(\Delta_1 \dashv A, \cdots, \Delta_n \dashv A)$ because $F(\Delta_1, \cdots, \Delta_n)$ includes B while $F(\Delta_1 \dashv A, \cdots, \Delta_n \dashv A)$ does not. This contradicts the assumption that F is dynamically rational. □

In other words, no uniform quota rule whose threshold is below the unanimity threshold n is dynamically rational with respect to any regular completeness-preserving operator. It could be of interest to the reader as to why Theorem 5 is applicable to a particular argumentation framework. It should be noted that this theorem is not restricted to some argumentation frameworks, but rather extends to a broad category of such frameworks, for instance, the argumentation frameworks that contain such arguments and attacks as depicted in Figure 1. Thus, our theorem is applicable to a wide range of argumentation frameworks.

A *winning coalition* $\mathcal{W} \subseteq N$ is a set of agents who can decide whether to accept or reject a given argument A. Given an aggregation rule F, if F is *neutral* and *independent*, then F can be fully determined by a single set $\mathcal{W}$ of winning coalitions, i.e., for every profile $\boldsymbol{\Delta}$ and every argument A it is the case that $A \in F(\boldsymbol{\Delta}) \Leftrightarrow N_A^{\boldsymbol{\Delta}} \in \mathcal{W}$.

Lemma 6. *Let F be an independent and neutral aggregation rule and let $\mathcal{W}$ be the corresponding set of winning coalitions for arguments. Then F is oligarchic if and only if $\mathcal{W}$ satisfies the following conditions:*

(i) *$\emptyset$ is not winning.*

(ii) *$C_1 \in \mathcal{W}$ implies $C_2 \in \mathcal{W}$ for any set $C_2 \subseteq N$ with $C_2 \supset C_1$.*

(iii) *For any pair of sets $\mathcal{C}, \mathcal{C}' \subseteq N$, $\mathcal{C}, \mathcal{C}' \in \mathcal{W}$ implies $\mathcal{C} \cap \mathcal{C}' \in \mathcal{W}$.*

Proof. Recall that F being oligarchic means that there exists a nonempty coalition $\mathcal{C}$ such that a given argument is accepted if and only if all the agents in $\mathcal{C}$ accept it. Thus, the winning coalitions are exactly $\mathcal{C}$ and its supersets. This family of sets does not include the empty set and is closed under both intersection and supersets.

Suppose F satisfies condition (i) to (iii). Let $\mathcal{C}^* := \bigcup_{\mathcal{C} \in \mathcal{C}} \mathcal{C}$, which is well-defined due to N being finite. Observe that $\mathcal{C}^*$ must be nonempty, due to the first three conditions. Now note that F is oligarchic with respect to coalition $\mathcal{C}^*$. □

Recall that an aggregation rule is said to be oligarchic if there exists a set of agents (a decisive set) whose individual extensions determine the outcome of the aggregation, which is the same as the intersection of their individual extensions. It is worth noting that a *filter* is a collection of subsets of N that satisfies the three properties specified in Lemma 6. The connection between filters and decisive sets is investigated in the literature of social choice theory[41, 15].

Lemma 7. *Let F be an independent and neutral aggregation rule and let $\mathcal{W}$ be the corresponding set of winning coalitions for arguments. Then F is dictatorial if and only if $\mathcal{W}$ satisfies the following conditions:*

(i) *$\emptyset$ is not winning.*

(*ii*) *For any pair of sets* $\mathcal{C}, \mathcal{C}' \subseteq N$, $\mathcal{C}, \mathcal{C}' \in \mathcal{W}$ *implies* $\mathcal{C} \cap \mathcal{C}' \in \mathcal{W}$.

(*iii*) *For any set* $\mathcal{C} \subseteq N$, $\mathcal{C}$ *or* $N \backslash \mathcal{C}$ *is in* $\mathcal{W}$.

Proof. F being dictatorial means that there exists an $i \in N$ such that the winning coalitions for argument are exactly $\{i\}$ and its supersets. This family of sets does not include the empty set, is closed under intersection, and maximal.

Suppose F is determined by Condition (*i*) to (*iii*) as far as arguments are concerned. Take an arbitrary $\mathcal{C} \in \mathcal{W}$ with $\mid \mathcal{C} \mid > 2$ and consider any nonempty $\mathcal{C}' \subsetneq \mathcal{C}$. By maximality, one of $\mathcal{C}$ and $N \backslash \mathcal{C}'$ must be in $\mathcal{W}$. Thus, by closure under intersection, one of $\mathcal{C} \cap \mathcal{C}' = \mathcal{C}'$ and $\mathcal{C} \backslash (N \backslash \mathcal{C}') = \mathcal{C} \backslash \mathcal{C}'$ must be in $\mathcal{W}$ as well. Observe that both of these sets are nonempty and of lower cardinality than $\mathcal{C}$. To summarise, we have just shown for any $\mathcal{C} \in \mathcal{W}$ with $\mid \mathcal{C} \mid > 2$ at least one nonempty proper subset of $\mathcal{C}$ is also in $\mathcal{W}$. By maximality, $\mathcal{W}$ is not empty. So take any $\mathcal{C} \in \mathcal{W}$. Due to N being finite, we can apply our reduction rule a finite number of times to infer that $\mathcal{W}$ must include some singleton $\{i\} \subsetneq \cdots \subsetneq \mathcal{C}$. Hence, F is a dictatorship with dictator i. □

It is also worth noting that an *ultrafilter* is a collection of subsets of N that satisfies the three properties specified in Lemma 7. As a consequence, given an aggregation rule F that is independent and neutral and let $\mathcal{W}$ be the corresponding set of winning coalitions for arguments, i.e., $A \in F(\boldsymbol{\Delta}) \Leftrightarrow N_A^{\boldsymbol{\Delta}} \in \mathcal{W}$ for all $A \in \Delta$. Then, F is dictatorial if and only if $\mathcal{W}$ is an ultrafilter.

Theorem 8. *Any grounded, monotonic, independent, and neutral aggregation rule that is dynamically rational with respect to any regular completeness-preserving revision operator for some argumentation frameworks must be oligarchic.*

Proof. For the sake of contradiction, assume that there is a non-oligarchic aggregation rule F satisfying all mentioned conditions and is dynamically rational with respect to a given regular completeness-preserving revision operator. Note that a completeness-preserving revision operator is also conflict-freeness-preserving. By Theorem 5, F guarantees conflict-freeness.

Let us consider the argumentation framework depicted in Figure 1. Note that $\{A, B, C\}$ is not complete. For each $Y \in \{A, B, C\}$, fix an extension $\Delta_{\neg Y}$ such that $\{A, B, C\} \backslash Y \subseteq \Delta_{\neg Y}$. Choose A from $\{A, B, C\}$ and note that since $\Delta_{\neg A} \dashv A$ cannot contain all $Y \in \{A, B, C\}$ (as the revision operator preserves completeness) but contains A (as the revision operator is successful), there exists some argument $X \in \{A, B, C\} \backslash \{A\}$ such that $X \notin \Delta_{\neg A} \dashv A$. Without loss of generality, assume that $X = B$, i.e., $B \notin \Delta_{\neg A} \dashv A$. Since $\mid \{A, B, C\} \mid \geqslant 3$, we can select the third argument in $\{A, B, C\} \backslash \{A, B\}$, i.e., select C.

By the assumption that F is independent and neutral, it can be described by its winning coalitions. Here are some points to prove:

(*i*) $\emptyset$ is not winning, this follows from the assumption that F is grounded.

(*ii*) Supersets of winning coalitions are also winning, meaning that if $\mathcal{C} \subseteq N$ is a winning coalition, then any superset $\mathcal{C}' \subseteq N$ containing $\mathcal{C}$ is also a winning coalition, by monotonicity of F.

(*iii*) Two winning coalitions $\mathcal{C}$ and $\mathcal{C}'$ must have at least one common argument; if not, then $N \backslash \mathcal{C} \supseteq \mathcal{C}'$ would be a winning coalition, which would result in two complementary winning coalitions and violate conflict-freeness.

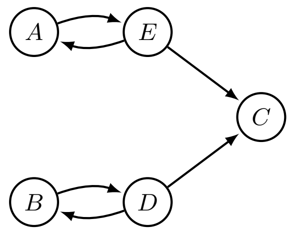

Figure 2: Argumentation framework with five arguments.

(iv) For any pair of sets $\mathcal{C}, \mathcal{C}' \subseteq N$, $\mathcal{C}, \mathcal{C}' \in \mathcal{W}$ implies $\mathcal{C} \cap \mathcal{C}' \notin \mathcal{W}$. Otherwise, the set of winning coalitions would be a filter over N, implying oligarchy by Lemma 6. Create a profile $(\Delta_1, \cdots, \Delta_n)$ where individuals in $N \backslash \mathcal{C}$ propose $\Delta_{\neg A}$, individuals in $\mathcal{C} \backslash \mathcal{C}'$ propose $\Delta_{\neg B}$, and individuals in $\mathcal{C} \cap \mathcal{C}'$ propose $\Delta_{\neg C}$. Since A and B are accepted by winning coalitions (specifically, by $\mathcal{C}$ and by $N \backslash (\mathcal{C} \backslash \mathcal{C}') \supseteq \mathcal{C}'$, respectively), A and B belong to $F(\Delta_1, \cdots, \Delta_n)$. Now, consider the revised profile $(\Delta_1 \dashv A, \cdots, \Delta_n \dashv A)$. In the new profile, B is only accepted by those individuals who submitted $\Delta_{\neg C}$ previously, as $B \notin \Delta_{\neg A} \dashv A$, and $\Delta_{\neg B} \dashv A$ and $\Delta_{\neg C} \dashv A$ are equivalent to $\Delta_{\neg B}$ due to the conservativeness of revision. Therefore, B is only accepted by the coalition $\mathcal{C} \cap \mathcal{C}'$. However, this coalition is not winning since it is a strict subset of a minimal winning coalition, either $\mathcal{C}$ or $\mathcal{C}'$. Thus, B does not belong to the set $F(\Delta_1 \dashv A, \cdots, \Delta_n \dashv A)$. Since $F(\Delta_1, \cdots, \Delta_n)$ contains A and as revision is conservative, it is equal to $F(\Delta_1, \cdots, \Delta_n) \dashv A$. However, $F(\Delta_1, \cdots, \Delta_n)$ and $F(\Delta_1 \dashv A, \cdots, \Delta_n \dashv A)$ differ because the former contains B while the latter does not. Therefore, $F(\Delta_1 \dashv A, \cdots, \Delta_n \dashv A) \neq F(\Delta_1, \cdots, \Delta_n) \dashv A$.

$\square$

Like Theorem 5, this theorem is widely applicable despite its assertion being limited to certain argumentation frameworks. Clearly, uniform quota rules are monotonic, independent, and neutral. In fact, Theorem 8 is a generalization of Theorem 5. If we hope to ensure the result of aggregation is a complete extension - a semantic property both fair and moderate - then we can further tighten this impossibility result and obtain a dictatorship.

Theorem 9. *Any grounded, independent, and neutral aggregation rule that is dynamically rational with respect to any regular completeness-preserving revision operator and preserves completeness for some argumentation frameworks must be dictatorial.*

Proof. For the sake of contradiction, assume that there is a non-dictatorial aggregation rule F that is grounded, independent, neutral, dynamically rational with respect to any regular completeness-preserving operator, and preserves completeness. Note that a completeness-preserving revision operator is also conflict-freeness-preserving. By Theorem 5, F guarantees conflict-freeness.

Let us consider the argumentation framework depicted in Figure 1. Note that $\{A, B, C\}$ is not complete. For each $Y \in \{A, B, C\}$, fix an extension $\Delta_{\neg Y}$ such that $\{A, B, C\} \backslash Y \subseteq \Delta_{\neg Y}$. Choose A from $\{A, B, C\}$ and note that since $\Delta_{\neg A} \dashv A$ cannot contain all $Y \in \{A, B, C\}$ (as the revision operator preserves completeness) but contains A (as the revision operator is successful), there exists some argument $X \in \{A, B, C\} \backslash \{A\}$ such that $X \notin \Delta_{\neg A} \dashv A$. Without loss of generality, assume that $X = B$, i.e., $B \notin \Delta_{\neg A} \dashv A$. Given that the cardinality of the set $\{A, B, C\}$ is greater than or equal to 3, we can pick the third argument in $\{A, B, C\} \backslash \{A, B\}$, i.e., pick C.

By the assumption that F is independent and neutral, it can be described by its winning coalitions. Here are some points to prove:

(*i*) $\emptyset$ is not winning, this follows from the assumption that F is grounded.

(*ii*) Two winning coalitions $\mathcal{C}$ and $\mathcal{C}'$ must have at least one common argument; if not, then $N \backslash \mathcal{C} \supseteq \mathcal{C}'$ would be a winning coalition, which would result in two complementary winning coalitions and violate conflict-freeness.

(*iii*) Given a set of agents $\mathcal{C}$ and its complement $N \backslash \mathcal{C}$, it must be the case that one of $\mathcal{C}$ and its complement $N \backslash \mathcal{C}$ is a winning coalition. Consider the argumentation framework depicted in Figure 3. Consider a profile in which agents in $\mathcal{C}$ propose $\{F, I\}$ and agents in $N \backslash \mathcal{C}$ propose $\{G, I\}$. Given that neither $\mathcal{C}$ nor $N \backslash \mathcal{C}$ constitutes a winning coalition, the aggregation rule will reject both G and F, which are the arguments exclusively supported by $\mathcal{C}$ and $N \backslash \mathcal{C}$ respectively. Furthermore, I is supported by all agents and N is a winning coalition, then $I \in F(\mathbf{\Delta})$. Then we get a collective set $\{I\}$ and violate completeness.

(*iv*) For any pair of sets $\mathcal{C}, \mathcal{C}' \subseteq N$, $\mathcal{C}, \mathcal{C}' \in \mathcal{W}$ implies $\mathcal{C} \cap \mathcal{C}' \notin \mathcal{W}$. Otherwise, the set of winning coalitions would be an ultrafilter over N, implying dictatorships by Lemma 7. Create a profile $(\Delta_1, \cdots, \Delta_n)$ where individuals in $N \backslash \mathcal{C}$ propose $\Delta_{\neg A}$, individuals in $\mathcal{C} \backslash \mathcal{C}'$ propose $\Delta_{\neg B}$, and individuals in $\mathcal{C} \cap \mathcal{C}'$ propose $\Delta_{\neg C}$. Since A and B are accepted by winning coalitions (specifically, by $\mathcal{C}$ and by $N \backslash (\mathcal{C} \backslash \mathcal{C}') \supseteq \mathcal{C}'$, respectively), A and B belong to $F(\Delta_1, \cdots, \Delta_n)$. Now, consider the revised profile $(\Delta_1 \dashv A, \cdots, \Delta_n \dashv A)$. In the new profile, B is only accepted by those individuals who submitted $\Delta_{\neg C}$ previously, as $B \notin \Delta_{\neg A} \dashv A$, and $\Delta_{\neg B} \dashv A$ and $\Delta_{\neg C} \dashv A$ are equivalent to $\Delta_{\neg B}$ due to the conservativeness of revision. Therefore, B is only accepted by the coalition $\mathcal{C} \cap \mathcal{C}'$. However, this coalition is not winning since it is a strict subset of a minimal winning coalition, either $\mathcal{C}$ or $\mathcal{C}'$. Thus, B does not belong to the set $F(\Delta_1 \dashv A, \cdots, \Delta_n \dashv A)$. Since $F(\Delta_1, \cdots, \Delta_n)$ contains A and as revision is conservative, it is equal to $F(\Delta_1, \cdots, \Delta_n) \dashv A$. However, $F(\Delta_1, \cdots, \Delta_n)$ and $F(\Delta_1 \dashv A, \cdots, \Delta_n \dashv A)$ differ because the former contains B while the latter does not. Therefore, $F(\Delta_1 \dashv A, \cdots, \Delta_n \dashv A) \neq F(\Delta_1, \cdots, \Delta_n) \dashv A$.

□

Thus, the dilemma identified by Theorem 5 for completeness is not restricted to uniform quota rules, but extends to all aggregation rules satisfying four desirable conditions. One may observe that Theorem 9 necessitates the guarantee of completeness and the elimination of monotonicity. Consequently, Theorem 8 and Theorem 9 illustrate the interplay between dynamic rationality and preservation requirements.

Before we close this section, we would briefly like to look at the implications of requiring only static rationality. Static rationality states that when aggregating a profile $\mathbf{\Delta} = (\Delta_1, \ldots, \Delta_n)$ in which every individual extension Δ_i satisfies a given property δ, the output $F(\mathbf{\Delta})$ satisfies δ as well. This notion of the preservation of properties of extensions is formally defined in Definition 10.

Theorem 10. *Any grounded, unanimous, neutral, and independent aggregation rule that preserves completeness must be dictatorial for some argumentation framework.*

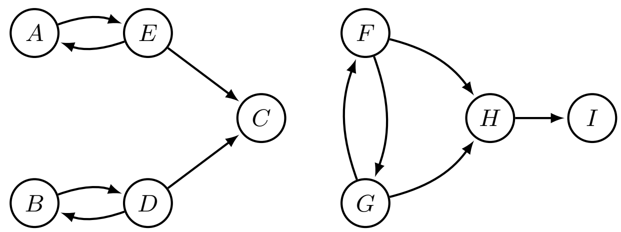

Figure 3: Argumentation framework used in the proof of Theorem 10.

Proof. Let $Arg = \{A, B, C, D, E, F, H, G, I\}$, let $\rightharpoonup$ be defined as in Figure 3, and let δ be the property of completeness. Now consider any aggregation rule F that is unanimous, grounded, neutral, and independent and that furthermore preserves δ. By the assumption that F is neutral and independent, F is determined by a collection of winning coalitions $\mathcal{W} \subseteq 2^N$, which, by Lemma 7, implies that the aggregation rule is dictatorial.

(*i*) As an immediate consequence of F being grounded, we obtain that $\emptyset \notin \mathcal{W}$.

(*ii*) To establish closure under intersection, take any two winning coalitions $C_1, C_2 \in \mathcal{W}$. Consider a profile in which exactly the individuals in C_1 accept A, exactly those in C_2 accept B, and exactly those in $C_1 \cap C_2$ accept C. Furthermore, suppose that nobody accepts any other argument. Observe that $\{A\}$, $\{B\}$, and $\{A, B, C\}$ all satisfy completeness by definition. Since C_1 and C_2 are winning coalitions, it also must include both A and B. Finally, by groundedness, it cannot include any argument outside of $\{A, B, C\}$. As outcome $\{A, B\}$ is ruled out by the requirement to preserve completeness, the outcome must include argument C. Hence, the coalition supporting C, which is $C_1 \cap C_2$, in fact, is a winning coalition. So we are done, as we have been able to show that $C_1, C_2 \in \mathcal{W}$ entails $(C_1 \cap C_2) \in \mathcal{W}$.

(*iii*) Let $\Delta = \{I\}$. To establish the maximality of $\mathcal{W}$, take any coalition $C \subseteq N$. Now consider a profile in which the individuals in C propose $\Delta \cup \{F\}$ and those in $N \setminus C$ propose $\Delta \cup \{G\}$. By unanimity, all arguments in Δ must be part of the outcome. Similarly, by groundedness, no argument outside of $\Delta \cup \{F, G\}$ can be part of the outcome. Due to F preserving completeness, the outcome must include at least one of F and G. If it includes F then its supporting coalition, C, is a winning coalition; if the outcome includes G then its supporting coalition, $N \setminus C$, is winning. In other words, we have been able to establish that either $C \in \mathcal{W}$ or $(N \setminus C) \in \mathcal{W}$, so $\mathcal{W}$ indeed is maximal.

□

Thus, even though we require static rationality only, completeness still cannot be preserved by aggregation rule satisfying appealing normative properties.

5. Contraction

In Section 4, we discussed the revision operator, which assumes that all unknown arguments (which act as new information) are acceptable. However, this assumption fails to capture situations

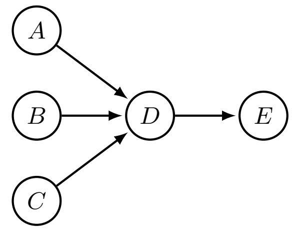

Figure 4: An argumentation framework with five arguments

where individuals receive information that an argument is unacceptable. To address this challenge, we introduce a new operator called the contraction operator.

Definition 20. *Given an extension $\Delta \subseteq Arg$ and an argument $A \in Arg$, a* **contraction operator** $\dotdiv$ *is a function that assigns a new extension $\Delta \dotdiv A$ to (Δ, A). We call a contraction operator* regular *if it satisfies the following two minimal conditions:*

(i) successful, i.e., $A \notin \Delta \dotdiv A$ for any pair (Δ, A);

(ii) conservative, i.e., $\Delta = \Delta \dotdiv A$ iff $A \notin \Delta$.

Thus, a contraction operator is successful if the learnt argument is excluded in the post-contraction extension; a contraction operator is conservative if the learnt argument is already excluded, then nothing changes.

In the following, we present an example that examines whether dynamic rationality is held when we consider excluding an unacceptable argument.

Example 3. Consider an argumentation framework $AF = \langle \{A, B, C, D, E\}, \{A \rightharpoonup D, B \rightharpoonup D, C \rightharpoonup D, D \rightharpoonup E\} \rangle$, as depicted in Figure 4. Let there be three agents, each of whom endorses a different admissible set of supported arguments, namely $\{A, E\}$, $\{B, E\}$, and $\{C, E\}$ for the first, second, and third agent, respectively, and who wish to preverve admissibility. By the majority rule, the outcome of the aggregation is $\{E\}$.

Suppose these agents learn that A is unacceptable. As a result, the first agent revises his extension to $\{B, E\}$ and rejects A, since accepting E violates the semantic property of admissibility. The individual extensions of the second and third agents are unaffected as they have already rejected A. Hence, the outcome under the majority rule becomes $\{B, E\}$. This scenario is depicted in Table 2.

On the other hand, if the agents aggregate their pre-revision extensions first, the resulting majority extension is $\{E\}$. After the revision in response to the unacceptability of A, the new extension remains as $\{E\}$, which differs from the pre-revision extension. △

Example 3 demonstrates that the minimal condition of successfulness and conservativeness of contraction operators is insufficient to guarantee dynamic rationality, and that the aggregation process preserves admissibility.

Definition 19 is about a revision operator, which is used to update an extension in light of a learnt argument, while Definition 20 is about a contraction operator, which is used to reduce an extension in light of a rejected argument. A revision operator is successful if the learnt argument is included in the new extension, while a contraction operator is successful if the rejected argument is excluded from the new extension.

	Pre-Learn. A				Post-Learn. A			
	A	B	C	E	A	B	C	E
Individual 1	✓	×	×	✓	×	✓	×	✓
Individual 2	×	✓	×	✓	×	✓	×	✓
Individual 3	×	×	✓	✓	×	×	✓	✓
Majority rule	×	×	×	✓	×	✓	×	✓

Table 2: Table used in Example 3

By Definition 16, a contraction operator $\vdash$ is referred to as an attacker-fixed contraction operator if, for a given set of arguments $\Delta \subseteq Arg$ and an argument $C \in Arg$, there exists a fixed attacker B for C such that no attacker A, who attacks B, can be found in $\Delta \vdash C$.

Proposition 11. *For any aggregation rule F, if F is grounded and is dynamically rational with respect to an attacker-fixed regular contraction operator, then F guarantees reinstatement.*

Proof. Let F be the specified aggregation rule, and let $\vdash$ be an attacker-fixed contraction operator. Consider a profile $\mathbf{\Delta} = (\Delta_1, \cdots, \Delta_n)$. Our goal is to show that $F(\mathbf{\Delta})$ is reinstating, which means that given an argument $C \in Arg$, if for every C's attacker B, there is an argument $A \in F(\mathbf{\Delta})$ such that $A \rightharpoonup B$ then $C \in F(\mathbf{\Delta})$.

Assume that for every C's attacker B, there is an argument $A \in F(\mathbf{\Delta})$ such that $A \rightharpoonup B$, for the sake of contradiction, we assume that $C \notin F(\mathbf{\Delta})$. By dynamic rationality of F, we have $F(\Delta_1 \vdash C, \cdots, \Delta_n \vdash C) = F(\Delta_1, \cdots, \Delta_n) \vdash C$. By conservativeness, we know that $F(\Delta_1, \cdots, \Delta_n) \vdash C = F(\Delta_1, \cdots, \Delta_n)$, so the right side of the equation equals $F(\Delta_1, \cdots, \Delta_n)$.

Assume that B is the attacker-fixed argument of C, then, for every B's attacker A, $A \notin \Delta_i \vdash C$ for all $i \in N$ (all B's attackers are excluded in $\Delta_i \vdash C$ for all $i \in N$). Therefore, $A \notin F(\Delta_1 \vdash C, \cdots, \Delta_n \vdash C)$ by the assumption that F is grounded. Hence, we conclude that $A \notin F(\Delta_1, \cdots, \Delta_n)$, which contradicts the assumption that $A \in F(\mathbf{\Delta})$.

If C has no attacker, then it will not be the case that C is defended by Δ but not included in Δ, i.e., this proposition holds trivially in this case. □

According to Definition 17, a contraction operator $\vdash$ is referred to as a two-attacker contraction operator when, provided with a set of arguments $\Delta \subseteq Arg$ and an argument $C \notin \Delta$, there exists a pair of attackers B_1, B_2 of C for which at least one of B_1 and B_2 has all of its attackers excluded in $\Delta \vdash C$.

Proposition 12. *Given an argumentation framework AF, if the majority rule F is dynamically rational with respect to a two-attacker regular contraction operator for AF and the number of agents is odd, then F preserves reinstatement for AF.*

Proof. Let F be the majority rule and let $\vdash$ be a two-attacker regular contraction operator. Let $\mathbf{\Delta} = (\Delta_1, \cdots, \Delta_n)$ be a profile with Δ_i is reinstating for all $i \in N$. We need to prove that $F(\mathbf{\Delta})$ is reinstating, meaning that given an argument $C \in Arg$, if C is defended by $F(\mathbf{\Delta})$, then $C \in F(\mathbf{\Delta})$.

Now we pick an argument $C \in Arg$, assume that for every C's attacker B, there is an argument $A \in F(\mathbf{\Delta})$ such that $A \rightharpoonup B$, we are going to show that $C \in \Delta$. For the sake of contradiction, we assume that $C \notin \Delta$. Given that F is dynamically rational, it follows that $F(\Delta_1 \vdash C, \cdots, \Delta_n \vdash C) = F(\Delta_1, \cdots, \Delta_n) \vdash C$. Moreover, due to the conservativeness of the contraction operator, we obtain $F(\Delta_1, \cdots, \Delta_n) \vdash C = F(\Delta_1, \cdots, \Delta_n)$.

If C has at least two attackers, suppose that B_1, B_2 are the two-attacker arguments of C, such that for all $i \in N$, either B_1 or B_2 have all their attackers excluded from $\Delta_i \vdash C$. Then, by the assumption that the number of agents is odd and applying the majority rule, we can infer that either B_1 or B_2 have all their attackers excluded from $F(\mathbf{\Delta} \vdash C)$. Hence, either B_1 or B_2 have all their attackers excluded from $F(\mathbf{\Delta})$, which leads to a contradiction.

If C has only one attacker, we denote it by B. By the assumption of all individual extensions are reinstating, every agent who accepts any of B's attackers need to accept C. Thus, $N_C^{\mathbf{\Delta}} \geqslant N_A^{\mathbf{\Delta}}$ for every B's attacker A. As a consequence, if $A \in F(\mathbf{\Delta})$, then $C \in F(\mathbf{\Delta})$ by the assumption that F is the majority rule.

If C has no attacker, then $C \in \Delta_i$ for all $i \in N$. Thus, $C \in F(\mathbf{\Delta})$, this completes the proof. □

By Definition 11, a contraction operator is self-defense-preserving if whenever Δ is self-defending, $\Delta \vdash A$ is self-defending for any extension $\Delta \subseteq Arg$ and any argument $A \in Arg$.

Fact 13. *The majority rule F is not dynamically rational with respect to a regular contraction self-defense-preserving operator for some argumentation frameworks.*

Proof. Let $AF = \langle \{A, B, C, D, E\}, \{A \rightharpoonup D, B \rightharpoonup D, C \rightharpoonup D, D \rightharpoonup E\} \rangle$. This argumentation framework is depicted in Figure 4. For the sake of contradiction, we assume that F is dynamically rational. Suppose we have a profile $\mathbf{\Delta} = (\{A, E\}, \{B, E\}, \{C, E\})$. Note that A, B, C are the three defenders of E and each agent supports one of them. Since $\{A, E\} \vdash A$ cannot exclude all of the defenders of E (as the contraction operator preserves self-defense), but excludes A (as the contraction operator is successful), there exists some $B \neq A$ such that $B \in \{A, E\} \vdash A$.

By the assumption that F is dynamically rational with respect to a regular contraction self-defense-preserving operator, we get that $F(\{A, E\} \vdash A, \{B, E\} \vdash A, \{C, E\} \vdash A) = F(\{A, E\}, \{B, E\}, \{C, E\}) \vdash A$. In this equation, the right side equals $F(\{A, E\}, \{B, E\}, \{C, E\})$ by conservativeness of the contraction operator and the fact that $A \notin F(\Delta_1', \cdots, \Delta_n')$ (A is only accepted by the first agent and thus not majoritarian supported). In the left side, since there are only three defenders, A, B, and C, but A is excluded ($A \notin \{A, E\} \vdash A$), and $B \in \{A, E\} \vdash A$, $\{B, E\} \vdash A = \{B, E\}$, and $\{C, E\} \vdash A = \{C, E\}$, it follows that there are at least two agents (a majority) who accept the same defender $B \neq A$.

Now, $F(\{A, E\}, \{B, E\}, \{C, E\})$ is equal to $F(\{A, E\}, \{B, E\}, \{C, E\}) \vdash A$ because it does not contain A and the contraction operator is regular. However, it differs from $F(\{A, E\} \vdash A, \{B, E\} \vdash A, \{C, E\} \vdash A)$ because $F(\{A, E\}, \{B, E\}, \{C, E\})$ excludes B while $F(\{A, E\} \vdash A, \{B, E\} \vdash A, \{C, E\} \vdash A)$ includes B. This contradicts the assumption that F is dynamically rational. □

Theorem 14. *No uniform quota rule F_q with $q > 1$ is dynamically rational with respect to any regular contraction self-defense-preserving operator for some argumentation frameworks.*

Proof. Let $AF = \langle\{A, B, C.D, E\}, \{A \rightharpoonup D, B \rightharpoonup D, C \rightharpoonup D, D \rightharpoonup E\}\rangle$ as illustracted in Figure 4. For the sake of contradiction, we assume that F is dynamically rational. Suppose we have a profile $\mathbf{\Delta}$. One agent supports $\{A, E\}$, $q-1$ agents supports $\{B, E\}$, and the remaining agents supports $\{C, E\}$. Note that A, B, C are the three defenders of E.

Since $\{A, E\}\dot{-}A$ cannot exclude all of the defenders of E (as the contraction operator preserves self-defense), but excludes A (as the contraction operator is successful), there exists some defender $B \neq A$ such that $B \in \{A, E\}\dot{-}A$.

By dynamic rationality $F(\Delta_1\dot{-}A, \cdots, \Delta_n\dot{-}A) = F(\Delta_1, \cdots, \Delta_n)\dot{-}A$. In this equation, the right side equals $F(\Delta_1, \cdots, \Delta_n)$ by conservativeness and the fact that $A \notin F(\Delta_1, \cdots, \Delta_n)$ (A is only accepted by the first agent and thus not accepted by q agents as $q > 1$). In the left side, since $B \in \{A, E\}\dot{-}A$, $\{B, E\}\dot{-}A = \{B, E\}$, and $\{C, E\}\dot{-}A = \{C, E\}$, it follows that B is accepted q agents. Therefore, $B \in F(\Delta_1\dot{-}A, \cdots, \Delta_n\dot{-}A)$.

Now, $F(\Delta_1, \cdots, \Delta_n)$ is equal to $F(\Delta_1, \cdots, \Delta_n)\dot{-}A$ as it does not contain A and the contraction operator is regular. However, it differs from $F(\Delta_1\dot{-}A, \cdots, \Delta_n\dot{-}A)$ because $F(\Delta_1, \cdots, \Delta_n)$ excludes B while $F(\Delta_1\dot{-}A, \cdots, \Delta_n\dot{-}A)$ includes B. This contradicts the assumption that F is dynamically rational. □

Theorem 5 and Theorem 14 are both about the dynamic rationality of uniform quota rules with respect to regular revision or contraction operators. They both show some impossibility results for such rules, meaning that they cannot satisfy certain desirable properties at the same time. However, Theorem 5 and Theorem 14 have different conditions on the quota q, different semantic properties (reinstatement vs. self-defense).

Now we present a theorem stating that dynamic rationality with respect to any regular contraction operator and the present five conditions cannot be satisfied together.

Theorem 15. *Any grounded, unanimous, independent, and neutral aggregation rule that is dynamically rational with respect to any regular contraction admissibility-preserving operator and guarantees admissibility for some argumentation frameworks must be dictatorial.*

Proof. For the sake of contradiction, assume that there is a non-dictatorial aggregation rule F satisfying all mentioned conditions and is dynamically rational with respect to a given regular contraction operator.

Consider the argumentation framework $AF = \langle\{A, B, C, D, E\}, \{A \rightharpoonup D, B \rightharpoonup D, C \rightharpoonup D, D \rightharpoonup E\}\rangle$ depicted in Figure 4. Note that $\{E\}$ is not admissible. For each $Y \in \{A, B, C\}$, fix an extension $\{E\} \cup \{Y\}$. Since $\{A, E\}\dot{-}A$ cannot exclude all of defenders of E (as the contraction operator preserves admissibility), but excludes A (as the contraction operator is successful), there exists some defender $B \neq A$ such that $B \in \{A, E\}\dot{-}A$.

By the assumption that F is independent and neutral, it can be described by its winning coalitions. Here are some points to prove:

(*i*) $\emptyset$ is not winning due to the assumption of F being grounded.

(*ii*) Two winning coalitions $\mathcal{C}$ and $\mathcal{C}'$ must have at least one common argument; if not, then $N \backslash \mathcal{C} \supseteq \mathcal{C}'$ would be a winning coalition, which would result in two complementary winning coalitions and violate conflict-freeness.

(*iii*) Given a set of agents $\mathcal{C}$ and its complement $N\backslash\mathcal{C}$, it must be the case that one of $\mathcal{C}$ and its complement $N\backslash\mathcal{C}$ is a winning coalition. Consider the argumentation framework depicted in Figure 4, consider a profile in which agents in $\mathcal{C}$ propose $\{A, E\}$ and agents in $N\backslash\mathcal{C}$ propose $\{B, E\}$. Due to unanimity, E is accepted by the aggregation rule. For the sake of contradiction, assume that both $\mathcal{C}$ and $N\backslash\mathcal{C}$ are not winning coalitions. Note that exactly agents in $\mathcal{C}$ support A and exactly agents in $N\backslash\mathcal{C}$ support B. By the assumption that none of $\mathcal{C}$ and $N\backslash\mathcal{C}$ is a winning coalition, both A and B are rejected by the aggregation rule. Due to groundedness, none of arguments other than A, B, and E can be included in $F(\boldsymbol{\Delta})$, which would result in a collective set $\{E\}$ and violate admissibility.

(*vi*) There are always at least two minimal winning coalitions; if this is not the case, then the set of winning coalitions would form an ultrafilter over N, which would imply dictatorship. Choose two different minimal winning coalitions, $\mathcal{C}$ and $\mathcal{C}'$. Create a profile $(\Delta_1, \cdots, \Delta_n)$ where individuals in $N\backslash\mathcal{C}'$ propose $\{E\} \cup \{A\}$, individuals in $C'\backslash C$ propose $\{E\} \cup \{C\}$, and individuals in $\mathcal{C} \cap \mathcal{C}'$ propose $\{E\} \cup \{B\}$. Given that only agents in $\mathcal{C} \cap \mathcal{C}'$ accepts B and $\mathcal{C} \cap \mathcal{C}'$ is not a winning coalition, as it is a strict subset of either $\mathcal{C}$ or $\mathcal{C}'$, it can be concluded that B is not part of the set $F(\Delta_1, \cdots, \Delta_n)$.

Now, consider the revised profile $(\Delta_1 \vdash A, \cdots, \Delta_n \vdash A)$. In the new profile, B is only accepted by those individuals in $N\backslash(\mathcal{C}'\backslash\mathcal{C})$, as $B \in \{E\} \cup \{A\} \vdash A$ and $N\backslash(\mathcal{C}'\backslash\mathcal{C})$ is a superset of winning coalition $\mathcal{C}$, and $\{E\}\cup\{B\} \vdash A$ and $\{E\}\cup\{C\} \vdash A$ are equivalent to $\{E\}\cup\{B\}$ and $\{E\}\cup\{C\}$ due to the conservativeness of the contraction operator. Thus, B belongs to the set $F(\Delta_1 \vdash A, \cdots, \Delta_n \vdash A)$.

As exactly the individuals in $N\backslash\mathcal{C}'$ propose $\{E\} \cup \{A\}$ and $N\backslash\mathcal{C}'$ in a complement of winning coalition $\mathcal{C}'$, $A \notin F(\Delta_1, \cdots, \Delta_n)$. Since $F(\Delta_1, \cdots, \Delta_n)$ does not contain A and as the contraction operator is conservative, it is equal to $F(\Delta_1, \cdots, \Delta_n) \vdash A$. However, $F(\Delta_1, \cdots, \Delta_n)$ and $F(\Delta_1 \vdash A, \cdots, \Delta_n \vdash A)$ differ because the former does not contain B while the latter does. Therefore, $F(\Delta_1 \vdash A, \cdots, \Delta_n \vdash A) \neq F(\Delta_1, \cdots, \Delta_n) \vdash A$.

□

Thus, dynamic rationality and admissibility cannot be guaranteed at the same time, at least when aggregation rules under examination satisfy certain fair conditions. According to Theorem 15, the issue highlighted in Theorem 14 is not limited to uniform quota rules but rather applies to all aggregation rules that meet the specified conditions. Furthermore, given that admissibility is a crucial element of various semantic properties, the infeasibility has significant consequences.

Theorem 16. *Any grounded, unanimous, independent, and neutral aggregation rule that is dynamically rational with respect to any regular contraction completeness-preserving operator and guarantees completeness for some argumentation frameworks must be dictatorial.*

Proof. For the sake of contradiction, assume that there is a non-dictatorial aggregation rule F satisfying all mentioned conditions and is dynamically rational with respect to a given regular contraction operator.

Consider the argumentation framework $AF = \langle\{A, B, C, A', B', C', D\}, \{A \rightharpoonup A', A' \rightharpoonup A, B \rightharpoonup B', B' \rightharpoonup B, C \rightharpoonup C', C' \rightharpoonup C, A' \rightharpoonup D, B' \rightharpoonup D, C' \rightharpoonup D\}\rangle$ depicted in Figure 1. Note that $\{D\}$ is not complete. For each $Y \in \{A, B, C\}$, fix an extension $\{D\} \cup \{Y\}$. Since $\{A, D\} \vdash A$ cannot exclude all of defenders of D (as the contraction operator preserves completeness), but excludes A (as the contraction operator is successful), there exists some defender $B \neq A$ such that $B \in \{A, D\} \vdash A$.

By the assumption that F is independent and neutral, it can be described by its winning coalitions. Here are some points to prove:

(i) $\emptyset$ is not winning due to the assumption of F being grounded.

(ii) Two winning coalitions $\mathcal{C}$ and $\mathcal{C}'$ must have at least one common argument; if not, then $N \backslash \mathcal{C} \supseteq \mathcal{C}'$ would be a winning coalition, which would result in two complementary winning coalitions and violate conflict-freeness.

(iii) Given a set of agents $\mathcal{C}$ and its complement $N \backslash \mathcal{C}$, it is necessary that either $\mathcal{C}$ or $N \backslash \mathcal{C}$ constitutes a winning coalition. Let us consider the argumentation framework illustrated in Figure 1 and consider a profile in which agents in $\mathcal{C}$ propose $\{A, D\}$ while agents in $N \backslash \mathcal{C}$ propose $\{B, D\}$. By virtue of unanimity, the aggregation rule accepts argument D. For the sake of contradiction, assume that both $\mathcal{C}$ and $N \backslash \mathcal{C}$ are not winning coalitions. Note that exactly agent in $\mathcal{C}$ support A and exactly agents in $N \backslash \mathcal{C}$ support B. By the assumption that none of $\mathcal{C}$ and $N \backslash \mathcal{C}$ is a winning coalition, both A and B are rejected by the aggregation rule. Due to groundedness, arguments other than A, B, and D cannot be included in $F(\boldsymbol{\Delta})$. However, this would result in a collective set $\{D\}$ and violate completeness.

(vi) There are always at least two minimal winning coalitions. If this condition is not met, the set of winning coalitions would form an ultrafilter over N, leading to a dictatorship. Let $\mathcal{C}$ and $\mathcal{C}'$ be two distinct minimal winning coalitions. We construct a profile $(\Delta_1, \cdots, \Delta_n)$ as follows: individuals in $N \backslash \mathcal{C}'$ propose $\{D\} \cup \{A\}$, individuals in $C' \backslash C$ propose $\{D\} \cup \{C\}$, and individuals in $\mathcal{C} \cap \mathcal{C}'$ propose $\{D\} \cup \{B\}$.

Considering that only agents in $\mathcal{C} \cap \mathcal{C}'$ accept B, and $\mathcal{C} \cap \mathcal{C}'$ is not a winning coalition as it is a strict subset of either $\mathcal{C}$ or $\mathcal{C}'$, we can conclude that B is not part of the set $F(\Delta_1, \cdots, \Delta_n)$.

Now, let us consider the revised profile $(\Delta_1 \vdash A, \cdots, \Delta_n \vdash A)$. In this new profile, B is only accepted by individuals in $N \backslash (\mathcal{C}' \backslash \mathcal{C})$ since $B \in \{D\} \cup \{A\} \vdash A$, and $N \backslash (\mathcal{C}' \backslash \mathcal{C})$ is a superset of the winning coalition $\mathcal{C}$. Also, $\{D\} \cup \{B\} \vdash A$ and $\{D\} \cup \{C\} \vdash A$ are equivalent to $\{D\} \cup \{B\}$ and $\{D\} \cup \{C\}$ due to the conservativeness of the contraction operator. Hence, B belongs to the set $F(\Delta_1 \vdash A, \cdots, \Delta_n \vdash A)$.

Since only individuals in $N \backslash \mathcal{C}'$ propose $\{D\} \cup \{A\}$ and $N \backslash \mathcal{C}'$ is the complement of the winning coalition $\mathcal{C}'$, it follows that $A \notin F(\Delta_1, \cdots, \Delta_n)$. As the contraction operator is conservative, $F(\Delta_1, \cdots, \Delta_n)$ is equal to $F(\Delta_1, \cdots, \Delta_n) \vdash A$. However, $F(\Delta_1, \cdots, \Delta_n)$ and $F(\Delta_1 \vdash A, \cdots, \Delta_n \vdash A)$ differ because the former does not contain B while the latter does. Therefore, $F(\Delta_1 \vdash A, \cdots, \Delta_n \vdash A) \neq F(\Delta_1, \cdots, \Delta_n) \vdash A$.

□

Theorem 16 states that the impossibilities identified in Section 4 for completeness (Theorem 5 and Theorem 8) are not restricted to revision operators, but extends to contraction operators.

Theorem 17. *Any grounded, unanimous, independent, and neutral aggregation rule that is dynamically rational with respect to either regular contraction preferredness- or stability-preserving operator* *and guarantees either preferredness or stability for some argumentation frameworks must be dictatorial.*

Proof. For the sake of contradiction, assume that there is a non-dictatorial aggregation rule F satisfying all mentioned conditions and is dynamically rational with respect to a given regular contraction operator.

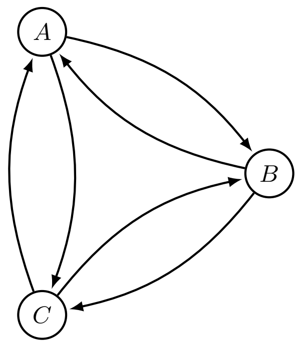

Figure 5: Scenario used in Theorem 17

Consider the argumentation framework $AF = \langle\{A, B, C\}, \{A \rightharpoonup B, B \rightharpoonup A, B \rightharpoonup C, C \rightharpoonup B, C \rightharpoonup A, A \rightharpoonup C\}\rangle$ depicted in Figure 5. Note that $\emptyset$ is neither preferred nor stable while all of $\{A\}$, $\{B\}$ and $\{C\}$ are preferred and stable. For each $Y \in \{A, B, C\}$, fix an extension $\{Y\}$. Since $\{A\}{\vdash}A$ cannot exclude all of A, B and C (as the contraction operator preserves preferredness or stability), but excludes A (as the contraction operator is successful), there exists some $B \neq A$ such that $B \in \{A\}{\vdash}A$.

By the assumption that F is independent and neutral, it can be described by its winning coalitions. Here are some points to prove:

- (*i*) $\emptyset$ is not winning due to the assumption of F being grounded.
- (*ii*) Two winning coalitions $\mathcal{C}$ and $\mathcal{C}'$ must have at least one common argument; if not, then $N \backslash \mathcal{C} \supseteq \mathcal{C}'$ would be a winning coalition, which would result in two complementary winning coalitions and violate conflict-freeness.
- (*iii*) Given a group of agents $\mathcal{C}$ and its complement $N \backslash \mathcal{C}$, it must be the case that one of $\mathcal{C}$ or its complement $N \backslash \mathcal{C}$ is a winning coalition. Let us examine the argumentation framework shown in Figure 5 and consider a profile in which agents in $\mathcal{C}$ propose $\{A\}$ while agents in $N \backslash \mathcal{C}$ propose $\{B\}$. Suppose, for the sake of contradiction, that both $\mathcal{C}$ and $N \backslash \mathcal{C}$ are not winning coalitions. Note that only agents in $\mathcal{C}$ support A and only agents in $N \backslash \mathcal{C}$ support B. According to our assumption, neither $\mathcal{C}$ nor $N \backslash \mathcal{C}$ is a winning coalition, resulting in the rejection of both A and B by the aggregation rule. Due to groundedness, no arguments other than A and B can be included in $F(\boldsymbol{\Delta})$, as this would lead to a collective set $\emptyset$ and violate preferredness and stability.
- (*vi*) There are always at least two minimal winning coalitions; if this condition is not met, then the collection of winning coalitions would form an ultrafilter over N, leading to a dictatorship. Select two distinct minimal winning coalitions, denoted as $\mathcal{C}$ and $\mathcal{C}'$. Construct a profile $(\Delta_1, \cdots, \Delta_n)$ as follows: individuals in $N \backslash \mathcal{C}'$ propose $\{A\}$, individuals in $\mathcal{C}' \backslash \mathcal{C}$ propose $\{C\}$, and individuals in the intersection of $\mathcal{C}$ and $\mathcal{C}'$ propose $\{B\}$.

Considering that only agents in $\mathcal{C} \cap \mathcal{C}'$ accept B, and since $\mathcal{C} \cap \mathcal{C}'$ is not a winning coalition as it is strictly contained within either $\mathcal{C}$ or $\mathcal{C}'$, it can be inferred that B is not part of the set $F(\Delta_1, \cdots, \Delta_n)$.

Now, let us examine the revised profile $(\Delta_1 \vdash A, \cdots, \Delta_n \vdash A)$. In this new profile, B is only accepted by individuals in $N \backslash (\mathcal{C}' \backslash \mathcal{C})$ since $B \in A \vdash A$ and $N \backslash (\mathcal{C}' \backslash \mathcal{C})$ is a superset of the winning coalition $\mathcal{C}$. Additionally, $\{B\} \vdash A$ and $\{C\} \vdash A$ are equivalent to $\{B\}$ and $\{C\}$ respectively due to the conservativeness of the contraction operator. Hence, B belongs to the set $F(\Delta_1 \vdash A, \cdots, \Delta_n \vdash A)$.

Since only individuals in $N \backslash \mathcal{C}'$ propose $\{A\}$ and $N \backslash \mathcal{C}'$ complements the winning coalition $\mathcal{C}'$, it can be deduced that $A \notin F(\Delta_1, \cdots, \Delta_n)$. As the contraction operator is conservative, $F(\Delta_1, \cdots, \Delta_n) \vdash A$ is equal to $F(\Delta_1, \cdots, \Delta_n)$ because the latter does not contain A. However, $F(\Delta_1, \cdots, \Delta_n)$ and $F(\Delta_1 \vdash A, \cdots, \Delta_n \vdash A)$ differ since the former does not include B while the latter does. Hence, $F(\Delta_1 \vdash A, \cdots, \Delta_n \vdash A) \neq F(\Delta_1, \cdots, \Delta_n) \vdash A$.

□

At this point, the preservation of almost all of the class semantic properties encounter difficulties when facing dynamic rationality, at least when the current operator is a contraction operator. This difficulty is so pervasive that we have to come up with a way to escape the impossibilities.

6. Dynamic rationality with restricted argumentation frameworks

The problem identified by impossibility theorems in previous sections is not restricted to a specific semantic property but extends to almost all practical and interesting properties. Thus, the impossibility applies very widely. In this section, we consider a possible escape route from the present impossibility results.

Central to the impossibility outcomes presented in this paper are three categories of conditions: the axioms governing aggregation rules, the semantic attributes of argumentation frameworks, and the specific argumentation frameworks under consideration. In response to these undesirable results, a potential strategy involves limiting the domain to encompass only permissible sets of argumentation frameworks. This idea is similar to the notion of domain restriction in the literature on preference aggregation [10, 40, 45] and judgment aggregation [27].

In this section, we study restricting the inputs to an aggregation rule such that only argumentation frameworks with a specific feature are allowed to be acted on by the aggregation rule.

Definition 21. *An argumentation framework is* **well-founded** *if and only if there exists no infinite sequence* $A_0, A_1, \cdots, A_n$ *of arguments such that for each* i, $A_{i+1} \rightharpoonup A_i$.

In the case of a finite argumentation framework, well-foundedness coincides with *acyclicity* of the attack-relations.

Theorem 18 (Dung, 1995)**.** *Every acyclic argumentation framework has exactly one complete extension that is grounded, preferred and stable.*

Proposition 19. *For any well-founded argumentation framework, every grounded aggregation rule* F *is dynamically rational with respect to any completeness-, preferredness-, or stability-preserving revision or contraction operator.*

Proof. Consider AF, an acyclic argumentation framework. It can be inferred from Theorem 18 that there exists only one extension within AF which is the only complete, preferred, and stable extension. This unique extension is denoted by Δ, while P denotes the properties of either completeness, preferredness, or stability. Let $\mid$ be an operator that is either a revision or contraction operator. It is clear that for any argument $A \in Arg$, $\Delta \mid A$ satisfies the property P. Take a profile $\boldsymbol{\Delta}$, since the choice for every individual can solely be Δ, the aggregation result of F is Δ. Consequently, the revision outcome of $F(\boldsymbol{\Delta})$ is Δ as well, i.e., $F(\boldsymbol{\Delta}) \mid A = \Delta$. Now, the revision result of every individual is Δ, i.e., $\Delta = \Delta \mid A$ for every argument $A \in Arg$, then $F(\Delta \mid A, \cdots, \Delta \mid A) = \Delta$. □

Theorem 20. *For a grounded aggregation rule F, if F is dynamically rational under any completeness, preferredness, or stability preserving revision or contraction operator, F guarantees completeness, preferredness, or stability for acyclic argumentation frameworks.*

Definition 22. *An argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$ is* **symmetric** *if for any pair of arguments $A, B \in Arg$, $A \rightharpoonup B$ if and only if $B \rightharpoonup A$.*

We also present a result concerning the relation between admissibility and conflict-freeness presented in Coste-Marquis et al. [19], which shows that admissible sets and conflict-free sets coincide in symmetric argumentation frameworks.

Proposition 21 (Coste-Marquis et al., 2005)**.** *Let $AF = \langle Arg, \rightharpoonup \rangle$ be a symmetric argumentation framework; then, a set of arguments $\Delta \in Arg$ is admissible if and only if it is conflict-free.*

In other words, an argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$ is symmetric if any argument A that attacks B is also counter-attacked by B. This paper specifically focuses on symmetric argumentation frameworks that do not have self-attacks, meaning that an argument cannot attack itself. However, some previous work include both types of symmetric argumentation frameworks: those that allow self-attacks and those that do not.

Fact 22. *Every regular revision or contraction operator is admissibility-preserving for symmetric argumentation frameworks.*

Proof. We consider an arbitrary symmetric argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$, an admissible set of arguments $\Delta \subseteq Arg$, and an argument $A \in Arg$. We also consider a revision operator, denoted by $\dashv$. If $A \in \Delta$, then nothing changes as the revision is conservative, i.e. $\Delta \dashv A$ is admissible. If $A \notin \Delta$, then $A \in \Delta \dashv A$ as the revision is successful. Suppose that there is an argument B such that $B \rightharpoonup A$, then according to the assumption that AF is symmetric: there is an attack $A \rightharpoonup B$, i.e., A defends itself, which means that $\Delta \dashv A$ is admissible.

Now, let us consider a contraction operator $\vdash$. If $A \notin \Delta$, then nothing changes as the contraction is conservative, i.e. $\Delta \vdash A$ is admissible. If $A \in \Delta$, then $A \notin \Delta \vdash A$ as the revision is successful. Take the argument $A' \in \Delta \vdash A$. For the sake of contradiction, assume that $\Delta \vdash A$ is not admissible, then suppose that there is an argument B such that $B \rightharpoonup A'$, then according to the assumption that AF is symmetric: there is an attack $A' \rightharpoonup B$, i.e., A' defends itself, which means that $\Delta \vdash A'$ is admissible. □

Proposition 23. *For any aggregation rule F, if F is grounded and dynamically rational with respect to a regular conflict-freeness-preserving revision operator, then F guarantees admissibility for symmetric argumentation frameworks.*

Proof. Let AF be the argumentation framework under consideration. Let F be an aggregation rule that is grounded. Suppose we have a profile $\mathbf{\Delta} = (\Delta_1, \cdots, \Delta_n)$, $A \in F(\mathbf{\Delta})$, and $A' \in Arg$ such that $A \rightharpoonup A'$. As AF is symmetric, if there is an argument B with $B \rightharpoonup A$, then $A \rightharpoonup B$ is the case, i.e., A defends itself. It remains to show that $A' \notin F(\mathbf{\Delta})$. By dynamic rationality, we have $F(\Delta_1 \dashv A, \cdots, \Delta_n \dashv A) = F(\Delta_1, \cdots, \Delta_n) \dashv A$. In this equation, the right side is equal to $F(\Delta_1, \cdots, \Delta_n)$ because of conservativeness and the fact that A is included in $F(\Delta_1, \cdots, \Delta_n)$. On the left side, since $A' \notin \Delta_i \dashv A$ for all $i \in N$, it follows from the groundedness of F that $A' \notin F(\Delta_1 \dashv A, \ldots, \Delta_n \dashv A)$. Consequently, we can conclude that A' is not included in $F(\mathbf{\Delta})$. This concludes the proof. □

7. Dynamic rationality with complete individual extensions

Judgment aggregation and extension aggregation are two ways of combining the opinions or choices of multiple agents into a collective outcome. Judgment aggregation [37] is a field of social choice theory that studies how to combine individual judgments on logically connected propositions into a collective judgment. For example, if a group of people has to decide whether to accept or reject a statement like "if it rains, then the grass is wet", they have to consider both the antecedent ("it rains") and the consequent ("the grass is wet") as well as the material implication between them.

One of the rationality requirements for judgment aggregation is completeness, which means that the set of accepted propositions must contain a member of every proposition-negation pair under consideration. In other words, for every proposition p, either p or its negation ($\neg p$) must be accepted. This ensures that the group does not abstain from making any judgment on the relevant issues.

However, completeness is not assumed to be satisfied by individual extensions, as an agent could reject all arguments, in this case, his individual extension is $\emptyset$. In this section, we consider the aggregation of individual extensions that satisfy completeness.

Before going any further, it is worth noting that, in line with the existing literature in argumentation theory on the one hand and judgment aggregation on the other, we use the term "complete" in two unrelated ways (complete extensions vs. complete individual extensions)

Building upon the notion of conflict-free sets of arguments in abstract argumentation frameworks, we now introduce a new concept that provides a more comprehensive understanding of the relationships between arguments. This concept, known as 'c-conflict-free,' extends the idea of conflict-free sets by considering both the presence of arguments involved in attacks and the absence of attacks within the set.

Definition 23. *An argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$, a set of arguments $\Delta \subseteq Arg$ is* **c-conflict-free** *if for every attack $A \rightharpoonup B$, either $A \in \Delta$ or $B \in \Delta$ and Δ is conflict-free.*

In a c-conflict-free set Δ, not only should there be no direct attacks between arguments within Δ, but also for every attack where one argument attacks another, at least one of the involved arguments should be a member of Δ. This additional condition ensures that the set captures a complete picture of the argumentative relationships, taking into account both the involved arguments and the attacks between them.

Theorem 24. *For any aggregation rule F, if F is unanimous, independent, neutral, and dynamically rational with respect to a conflict-freeness-preserving revision operator, then F guarantees conflict-freeness.*

Proof. Assume that F is the given aggregation rule. Let $\mathbf{\Delta} = (\Delta_1, \cdots, \Delta_n)$ be a profile and suppose that $A \in F(\mathbf{\Delta})$ and $A' \in Arg$ such that $A \rightharpoonup A'$. Our goal is to show that $A' \notin F(\mathbf{\Delta})$. To do so, we construct a set $\Delta \subseteq Arg$ that satisfies c-conflict-freeness and excludes B', and define a profile $\mathbf{\Delta'}$ as follows:

$\Delta'_i = \Delta \dashv B$ if $A \in \Delta_i$;

$\Delta'_i = \Delta$ if $A \notin \Delta_i$.

Since Δ satisfies c-conflict-freeness, $A \notin \Delta_i$ if and only if $A' \in \Delta_i$. Note that $A \in \Delta_i$ if and only if $B \in \Delta'_i$ and $A' \in \Delta_i$ if and only if $B' \notin \Delta'_i$. By independence and neutrality, it suffices to prove that $B' \notin F(\mathbf{\Delta'})$. By conflict-freeness-preservation, we know that $\langle \Delta'_1 \dashv B, \cdots, \Delta'_n \dashv B \rangle$ is in the domain of F. By dynamic rationality, we have $F(\Delta'_1 \dashv B, \cdots, \Delta'_n \dashv B) = F(\Delta'_1, \cdots, \Delta'_n) \dashv B$. In this equation, the left side equals $F(\Delta \dashv B, \cdots, \Delta \dashv B)$, which is equal to $\Delta \dashv B$ by unanimity preservation. The right side equal $F(\Delta'_1, \cdots, \Delta'_n)$ by conservativeness and the fact that $B \in F(\Delta'_1, \cdots, \Delta'_n)$. Therefore, we have $\Delta \dashv B = F(\mathbf{\Delta'})$ and $B' \notin F(\mathbf{\Delta'})$, which implies that $A' \notin F(\mathbf{\Delta})$. This completes the proof. □

Both Theorem 1 and Theorem 24 demonstrate the guarantee of conflict-freeness. They both share the property of dynamic rationality with respect to a conflict-freeness-preserving revision operator, but they differ in the additional properties that the aggregation rule F must possess. Theorem 1 stipulates that for the aggregation rule F to guarantee conflict-freeness, it needs to be grounded and dynamically rational with respect to a regular conflict-freeness-preserving revision operator. Theorem 24, on the other hand, requires the given aggregation rule F to possess more properties - it must be unanimous, independent, neutral, and dynamically rational with respect to a conflict-freeness-preserving revision operator.

8. Related work

The dynamic aspect of abstract argumentation has been widely studied (see [33] for an overview). One key area of interest is how changes occur in dynamic argumentative scenarios, or the consequences of the addition and the removal of an atomic element of argumentation frameworks. There are two main approaches to studying change in Dung's argumentation framework: elementary change [12, 8, 16, 39], which concerns that when elements changed, whether a given property is satisfied, and extension enforcement [4], which studies strategies for modifying argumentation frameworks to make sure that a specific set of arguments is part of its extension. In addition to change, constraints are also an important consideration in argumentation systems [16, 9, 31, 32, 20, 21]. Different kinds of constraints can be considered in an argumentation system, and their enforcement implies changes. What is more, a comprehensive analysis is conducted by Baumann and Penndorf [5] to evaluate argumentation frameworks based on information they possessed, specifically examining if one argumentation framework contains more information compared to another.

By contrast, the question of dynamic rationality has been considered in the literature on other branches of social choice theory. Dietrich and List [28, 29] study whether there are reasonable aggregation rules that enable a group to achieve dynamic rationality when aggregating interconnected propositional formulas. The question of dynamic rationality has also received much attention in the distinct setting of probability aggregation, where judgments aren't binary but take the form of subjective probability assignments to the elements of some algebra (e.g., Dietrich [26], Russell et al. [44]).

There are several work that combine belief revision and argumentation. Coste-Marquis et al. [22] introduce novel enforcement strategies for modifying argumentation frameworks to make sure that a specific set of arguments is part of its extension, which are close to the process of belief revision. Baroni et al. compare and relate belief revision and argumentation as approaches to model reasoning processes [3]. Coste-Marquis et al. [21] derive argumentation systems that satisfy given revision formulas, i.e., given an argumentation system and a revision formula that expresses how the statuses of some arguments have to be changed under a chosen semantics, the derived argumentation systems are such that the corresponding extensions are as close as possible to the extensions of the input system. Diller et al. [30] follow this work and study how to update an argumentation framework with new information, either a formula or another argumentation framework, and defines rationality criteria for such updates. Paglieri and Castelfranchi propose a data-oriented belief revision that enables incorporating computational argumentation [42]. Booth et al. [14] propose two methods to restore consistency when an agent's belief state, composed of a Dung's argumentation framework and a propositional constraint, are inconsistent: firstly, a normal expansion that may alter the set of complete labellings; and secondly, belief revision techniques that aim to retain similarity to the original labellings.

The literature on collective argumentation has extensively examined the aggregation of individual extensions into collective ones, primarily in static scenarios. Researchers have investigated whether semantic properties like conflict-freeness, admissibility, and completeness are preserved at a given point in time. Some notable work in this topic include Tohmé et al. [46], Dunne et al. [35], Chen and Endriss [18, 17], and Rahwan and Tohmé [43]. Despite this, the dynamic aspect of collective argumentation has been studied implicitly or explicitly. Bonzon and Maudet [13] concern an argument game in which every step follows a specific protocol. The solution of the game will determine the acceptance of a single argument under the assumption that agents share the same set of arguments but support different attack-relations between these arguments. Kontarinis et al. [38] consider dynamic systems that allow the addition and removal of attacks. They study the minimum changes required to achieve argument acceptance or non-acceptance, along with the properties of such changes. Delobelle et al. [23, 24, 25] explore the merging of abstract argumentation frameworks. This process involves two steps: selecting the set of revised candidates (a set of argumentation frameworks) and generating the argumentation frameworks that represent these candidates. They use distance-based merging operators to compute the generated argumentation framework. A comprehensive overview of research on collective argumentation beyond this small selection we refer to the survey by Bodanza et al. [11]. In this survey, the authors introduce two collective argumentation approaches, namely argument-wise and framework-wise approaches, and our paper falls under the argument-wise approach. This survey also presents the works on collective argumentation that use social choice methods, which our paper also employs.

9. Conclusion

In this paper, we have explored the concept of dynamic collective argumentation, which involves revising and aggregating individual argumentative stances in response to new information. We proposed the revision and contraction operator to handle different dynamic aspects of collective argumentation. We have focused on the preservation of semantic properties, such as conflict-freeness, reinstatement, and self-defense, when aggregation and revision commute, i.e., when they lead to the same outcome regardless of the order in which they are applied. We have shown some impossibility results for certain classes of aggregation rules. These results state that no rule belonging to the class satisfies these semantic properties and dynamic rationality simultaneously. The paper contributes to the literature on abstract argumentation by examining how new information can affect the rationality of collective decisions. It also provides insights into the interplay between individual and collective reasoning processes in dynamic argumentative scenarios.

There are several directions to extend and strengthen the current work. Currently, we only incorporate the revision operator from belief revision theory. However, this work can be expanded by integrating other belief revision operators, such as expansion. Belief expansion refers to the process of adding new beliefs without removing existing ones. Incorporating these additional operators would enable the model to account for a wider range of scenarios involving belief changes. Instead of defining the new information as a single learnt argument, this work can be generalized by considering the new information as a set of learnt arguments or information comprising both learnt accepted and rejected arguments. Representing individual argumentative stances as a mere set of arguments is a rather simplified assumption. A more sophisticated approach would be modeling individual argumentative stances as argumentation frameworks, i.e. each agent adopting a distinct set of attack-relations on the same set of arguments. The new information in this case would consist of some learnt attacks between arguments. Currently, we mainly focus on some basic properties of semantics, such as conflict-freeness, admissibility, and completeness. These semantics have the feature of being characterized by logical formulas. Despite these advantages, we should continue to study other semantics in the future, such as priority semantics and stable semantics.

Finally, we note that we have proposed several revision and contraction operators in this paper. In addition to basic successfulness and conservativeness, these operators also need to satisfy other properties, such as conflict-freeness-preserving and admissibility-preserving. Although these requirements are easy to meet, a noteworthy problem is that they sometimes cannot be satisfied simultaneously. For example, in an acyclic argumentation framework, if the revised argument is not in the unique extension, then admissibility-preservation and successfulness cannot be satisfied at the same time, because such an argument cannot belong to any extension. In future research, it would be interesting to investigate the conditions under which operators can simultaneously achieve successfulness, conservativeness, and semantic properties.

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