Collective Argumentation with Topological Restrictions: The Case of Aggregating Abstract Argumentation Frameworks

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Abstract

Collective argumentation is the process of reaching a collective decision that is acceptable to the group in a debate. We introduce the notion of topological restriction to enrich the study of collective argumentation. Topological restrictions are rational constraints assumed to be satisfied by individual agents. We assume that in a debate, for every pair of arguments under consideration, every agent indicates whether the first argument attacks the second, i.e., an agent's argumentative stance is characterized as an argumentation framework, and only argumentation frameworks that satisfy topological restrictions are allowed. The topological restrictions we consider in this paper include various topological properties in the literature, such as acyclicity, symmetry, coherence, and determinedness, as well as three topological restrictions that generalize classic social-choice-theoretic domain conditions. We show that when the profile of the argumentation frameworks provided by the agents satisfies topological restrictions, impossibility results during aggregation can be avoided. Furthermore, if a profile is topologically restricted with respect to restrictions that generalize domain conditions, then the majority rule preserves several desirable properties during aggregation.

1 Introduction

Argumentation has long been used to resolve differences of opinion. As a formalism that addresses the formalization of argumentation, abstract argumentation theory [29] has been applied for over twenty years to analyse argument justification. An abstract argumentation framework is simply a set of arguments on which a binary attack relation is defined. When there are several agents involved in a debate, such as a juridical or parliamentary debate, they may have different opinions on the evaluation of the acceptability of arguments or the justification of attacks between arguments. Collective argumentation has been discussed extensively in the literature on formal argumentation (see [14, 8]). Among related works, some are dedicated to investigating the aggregation of arguments [26, 49, 23, 21], while others study the aggregation of attacks [16, 44, 14, 22, 20]. In a broad sense, the aim of collective argumentation is to find consensuses that are acceptable among agents when they present either different sets of arguments or different attack-relations in an argumentation process.

The problem of aggregating abstract argumentation frameworks (AFs) has received attention in the literature in the last decade or so [26, 49, 23, 14]. In particular, techniques from *social choice* *theory* [2, 38, 15] have been widely used in the literature on collective argumentation. The *axiomatic method* [47, 48, 34] is one of the most important methods for studying aggregation. However, in research on the aggregation of argumentation frameworks using the axiomatic method, no restriction has been placed on the AFs proposed by individual agents, which is an important gap in the literature. For example, as mentioned by Chen and Endriss [23], majority voting with unrestricted but acyclic individual AFs may generate collective AFs that contain cycles, while on suitably restricted AFs, the semantic property of acyclicity can be preserved. Given three arguments A, B, and C, if one individual supports that $A \rightharpoonup B$ and $B \rightharpoonup C$, a second that $B \rightharpoonup C$ and $C \rightharpoonup A$, and a third that $A \rightharpoonup C$ and $C \rightharpoonup A$, then there are majorities for $A \rightharpoonup B$, $B \rightharpoonup C$, and $C \rightharpoonup A$, which form a cycle; however, if no individual supports $A \rightharpoonup B$, for example, acyclicity will be preserved.

Unsurprisingly, an abundance of impossibility results closely related to the famous Condorcet Paradox in the theory of preference aggregation [43] have been reported in the literature on collective argumentation. For example, Chen and Endriss show that only dictatorships, which are aggregation rules that are clearly unacceptable from an axiomatic point of view, can preserve the most demanding semantic properties [23]. Thomé et al. show that only aggregation rules that have "hidden dictators" will always preserve the property of acyclicity, a property that greatly simplifies the evaluation of arguments (in an acyclic argumentation framework, it is unambiguous which arguments to accept and which to reject) [49].

In the widely used system of abstract argumentation, each individual agent provides an arbitrary argumentation framework that represents her argumentative stance in a debate. In this case, we assume that for every pair of arguments under consideration in a debate, every agent indicates whether the first attacks the second. Given a semantic property agreed upon by the individual agents, the output may or may not satisfy this property.

We introduce a type of domain condition, called a topological restriction, that can help us eliminate argumentation frameworks that are not desirable. For example, we may consider it irrational for an individual agent to support argumentation frameworks that contain cycles. In this case, we can require that the agents' argumentation frameworks satisfy acyclicity. For acyclic argumentation frameworks, the acceptance status of arguments is unambiguous, as the grounded extension coincides with the unique preferred extension, which is also stable. Other topological properties also appear in the literature on abstract argumentation. If such properties are taken as conditions on individual argumentation frameworks in the sense that all individual AFs must satisfy such properties, then we call them topological restrictions.

Related work The problem of aggregating abstract argumentation frameworks has been studied extensively in the literature [23]. However, little is known about the role of topological restrictions during aggregation. The notion of topological restriction arises from that of domain restriction in the theory of preference aggregation and judgment aggregation. It is known that in preference aggregation, voting on pairs of alternatives with the majority rule may generate inconsistent (i.e., cyclical) collective preferences even if all individuals' preferences are consistent (i.e., acyclic). Domain restrictions have been used to resolve this dilemma, including Black's single-peakedness [10], Inada's single-cavedness [40], and the triplewise value restriction presented by Sen [45]. In judgment aggregation, the relevant domain restriction approaches include unidimensional alignment, as proposed by List [42], and value restriction, as proposed by Dietrich and List [28]. Colley [24] translates value restriction from judgment aggregation to binary aggregation.

In the literature on collective argumentation, however, domain restrictions have received much less attention. Chen [20] considers a similar notion called value restriction during the aggregation of extensions of argumentation frameworks, another scenario that has been studied extensively in the literature on collective argumentation [22, 16]. His focus is on the guarantee of admissibility, a property at the heart of most semantics of argumentation frameworks. Chen shows that if an argument profile is value-restricted with respect to a constraint in the sense that individual agents agree or disagree with the membership of a specific pair of arguments, then the majority rule guarantees feasible outcomes.

Now, let us briefly introduce the literature on the aggregation of argumentation frameworks. Coste-Marquis et al. [26] are the first to address the question of how best to aggregate several abstract argumentation frameworks. They focus on a family of sophisticated aggregation rules that minimise the distance between the input argumentation frameworks and the output argumentation framework. We build on this idea and focus on simple rules, particularly rules that satisfy the axiom of independence. Bodanza and Auday [13] are the first to give a completely general definition of an aggregation rule mapping any set of individual argumentation frameworks to a collective argumentation framework. Dunne et al. [33] define several preservation requirements on aggregation rules that directly refer to the semantics of the argumentation frameworks concerned. In follow-up work, Delobelle et al. [27] establish for several concrete rules whether they satisfy the preservation requirements introduced by Dunne et al. [33]. We build on the idea of integrating abstract argumentation and social choice theory and distinguish conditions for aggregation rules and preservation requirements referring to semantics. Thomé et al. [49] are the first to explicitly use social choice theory to analyse the aggregation of argumentation frameworks. Their focus is on the preservation of the acyclicity of attack-relations under aggregation. We also make use of techniques from social choice theory; however, we focus on a variety of properties rather than a single property. Regarding the literature on the generalization of Arrow's Impossibility Theorem, we recall the work by Chen and Endriss [23] and the work by Thomé et al. [49].

Bodanza et al. [14] present a survey of work on the aggregation of attack-relations or extensions of argumentation frameworks. A more specific overview of the aggregation of argumentation frameworks using methods from the social choice methodology can be found in the survey by Baumeister et al. [8]. An alternative notion of collective argumentation is defined by Bochman [11, 12], which postulates a primitive attack-relation that holds between sets of arguments. The idea of modelling attacks that involve sets of arguments has also been discussed extensively in the literature (e.g., Bikakis et al. [9], Gabbay [37, 36], Barringer et al. [7], Cayrol et al. [17]). Another work that involves multiagent collective argumentation is the work by Yu et al. [50], who propose abstract agent argumentation frameworks that extend Dung's abstract argumentation theory. Regarding the semantics of abstract agent argumentation frameworks, whether an argument is acceptable can also depend on the agents and the relations associating arguments with agents.

In this paper, we apply the so-called axiomatic method [47, 48, 34]. This method amounts to formulating axioms that are normatively desirable properties of aggregation rules to obtain characterisation results for the aggregation rules satisfying these axioms. Arrow's Impossibility Theorem [1] is a notable example of such a characterisation. This method is also adopted by Li [41], who analyses a variant of Sen's famous Paradox of the Paretian Liberal [46] in the context of aggregating argumentation frameworks. By comparison, Dunne et al. [33] define several preservation requirements on aggregation rules that directly refer to the semantics of the argumentation frameworks concerned. Their focus is on analysing the computational complexity of deciding whether a given aggregation rule has a given property, rather than on the axiomatic method. This work is followed by that of Delobelle et al. [27].

Contribution We introduce the notion of topological restriction for the aggregation of argumentation frameworks. Such restrictions are conditions on the profiles of attack-relations that preserve desirable semantic properties. We study the roles played by several topological restrictions during aggregation, including acyclicity, symmetry, coherence and determinedness, which are topological properties of AFs, as well as t-self-defense, t-attack and t-acy, which are generalizations of classical social-choice-theoretic domain conditions. We show that with topological restrictions, impossibility results can be avoided during the aggregation of attack relations, i.e., aggregation rules emerge that preserve desired properties. Furthermore, we show that if a profile is topologically restricted with respect to restrictions that generalize social-choice-theoretic domain conditions, then the majority rule—a rule that is highly appealing on normative grounds because it treats all agents in a "fair" manner—preserves several semantic properties during aggregation.

An earlier version of this paper has been presented at CLAR-2021 [19]. We have been able to significantly extend and strengthen the results presented in that earlier paper: (*i*) we now also cover the topological properties of coherence and determinedness (in addition to the properties of acyclicity and symmetry already covered in the earlier paper), (*ii*) we introduce the topological restrictions of t-attack and t-acy (in addition to the restriction of t-self-defense already introduced in the earlier paper), and (*iii*) we discuss the relation between value restriction in judgment aggregation and the topological restrictions studied in this paper. In addition, we have significantly expanded the discussion of related work and future work.

Paper overview The remainder of this paper is organized as follows. Section 2 presents relevant concepts from the theory of abstract argumentation, including some of the fundamentals of the model of abstract argumentation, topological properties, and semantics agreement. Section 3 introduces our model, and Section 4 introduces the concept of topological restriction. Section 5 studies four topological restrictions that are based on the well-known topological properties of acyclicity, symmetry, coherence, and determinedness and presents the preservation results obtained with such restrictions. Section 6 introduces three topological restrictions called t-self-defense, t-attack and t-acy, which generalize social-choice-theoretic domain restrictions, and presents preservation results obtained with these restrictions for several semantic properties. Section 7 discusses the relation between value restriction in judgment aggregation and the topological restrictions studied in this paper. Section 8 concludes the paper and suggests some directions for future work.

2 Argumentation framework and topological properties

An argumentation framework is a pair $AF = \langle Arg, \rightharpoonup \rangle$, in which Arg is a set of arguments and $\rightharpoonup$ is a set of pairs of arguments called the attack-relation built on Arg. Given two arguments $A, B \in Arg$, if $A \rightharpoonup B$ holds, we say that A attacks B. Given a set of arguments $\Delta \subseteq Arg$, we say that Δ is *conflict-free* if there are no arguments $A, B \in Arg$ such that $A \rightharpoonup B$; we say that Δ defends $A \in Arg$

if for every argument $B \in Arg$ such that $B \rightharpoonup A$, there is an argument $C \in \Delta$ such that $C \rightharpoonup B$; we say that Δ is self-defending if Δ defends every argument in Δ; and we say that Δ is *admissible* if Δ is conflict-free and self-defending. Furthermore, we say that:

- Δ is complete if Δ is admissible and every argument defended by Δ is included in Δ.
- Δ is grounded if Δ is the minimal complete extension (w.r.t. set inclusion).
- Δ is preferred if Δ is a maximal admissible set (w.r.t. set inclusion).
- Δ is stable if Δ is conflict-free and attacks every argument that is not in Δ.

In other words, a set of arguments is complete if it is admissible and every argument defended by it is a member of it. There is always exactly one grounded extension, which is a set of acceptable arguments that presents no ambiguity. A set of arguments is preferred if it is a maximal admissible set, meaning that adding any additional arguments to the set makes it no longer admissible. A set of arguments is stable if it attacks every argument that is not in the set and contains no internal conflicts.

A semantics defines which set of arguments can be accepted and can be considered a property of sets of arguments. We now present another family of properties considered in the literature on abstract argumentation, namely, the *topological properties* of argumentation frameworks. While the topological properties of argumentation frameworks have no immediately apparent relation with argumentation semantics, they play an important role in the study of such semantics. As early as the seminal paper by Dung [29], well-foundedness has been identified as a topological property and has been shown to be a sufficient condition for agreement among grounded, preferred, and stable semantics, namely, the grounded extension is the only preferred and stable extension.

Definition 1. *An argumentation framework is* well-founded *if and only if there exists no infinite sequence* $A_0, A_1, \cdots, A_n$ *of arguments such that for each* i*,* $A_{i+1} \rightharpoonup A_i$*.*

In the case of a finite argumentation framework, well-foundedness coincides with *acyclicity* of the attack-relations.

Definition 2. *An argumentation framework* $AF = \langle Arg, \rightharpoonup \rangle$ *is coherent if every preferred extension of* AF *is stable.*

The absence of odd-length cycles is a sufficient condition to ensure that an argumentation framework is coherent, i.e., that stable extensions exist and coincide with preferred extensions. Clearly, if an argumentation framework is acyclic, then it is coherent, i.e., the set of acyclic *AF*s is a subset of the set of coherent *AF*s.

Definition 3. *An argumentation framework* $AF = \langle Arg, \rightharpoonup \rangle$ *is a* symmetric *argumentation framework if* $\rightharpoonup$ *is symmetric, nonempty and irreflexive.*

Example 1. Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework in which $Arg = \{A, B, C, D, E\}$, $\rightharpoonup = \{A \rightharpoonup B, B \rightharpoonup A, B \rightharpoonup C, C \rightharpoonup B, C \rightharpoonup A, A \rightharpoonup C, C \rightharpoonup D, D \rightharpoonup C\}$, as illustrated in Figure 1. Clearly, AF is a symmetric argumentation framework. $\triangle$

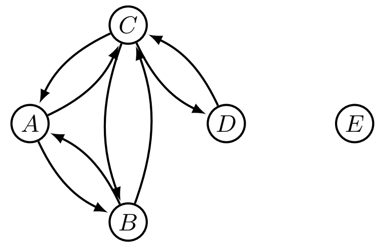

Figure 1: Scenarios used in Example 1.

In other words, an argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$ is symmetric if for any pair of arguments $A, B \in Arg$ with A attacks B, then B will be counter-attacked by A. In this paper we focus on symmetric AFs that are irreflexive, which means that no self-attacks exist. However, some works in the literature consider both cases, i.e., consider symmetric AFs where self-attacks are authorized and symmetric AFs where self-attacks are forbidden.

Definition 4. *An argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$ is* determined *if and only if for every argument $A \in Arg$ that is not included in the grounded extension of AF, A is attacked by at least one argument in the grounded extension.*

In other words, an argumentation framework AF is determined if and only if the grounded extension of AF is also a stable extension. Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework in which $Arg = \{A, B, C\}, \rightharpoonup = \{A \rightharpoonup B, B \rightharpoonup C, C \rightharpoonup B\}$. It is easy to verify that AF is a determined argumentation framework, as B is attacked by A and C, two members of the grounded extension of AF. It is worth noting that acyclic argumentation frameworks are a special case of determined argumentation frameworks.

Other topological properties of argumentation frameworks in the literature include *directionality*, introduced in [3]; *SCC-symmetry*, introduced in [5]; *almost determinedness*, introduced in [4]; and *limited controversy*, introduced by Dung in his seminal work [30]. For a detailed overview of topological properties beyond this small selection, we refer to the survey by Baroni et al. [6].

We now recall the notion of *semantics agreement* discussed in the literature [4]. Given an AF, if two semantics prescribe the same set of extensions, then we say that these semantics are in agreement (with respect to AF). Given a topological property, we say that two semantics are in agreement if they prescribe the same sets of extensions in any argumentation framework satisfying this topological property. One example is acyclicity: for every acyclic argumentation framework, the complete, preferred, and stable extensions coincide. Thus, one interesting question to consider is what topological properties will lead semantics to be in agreement. Because the problem we consider in this paper is collective argumentation, if two semantics are in agreement, then the choice of semantics turns out to have no influence, and the collective outcomes are semantics-independent.

3 The model

Given a set of arguments Arg and a set of agents $N = \{1, \cdots, n\}$, suppose that each agent provides an argumentation framework reflecting her individual views on the statuses of possible attacks between arguments. Thus, we are given a profile of attack-relations $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$. Sometimes, we may wish to aggregate individual argumentation frameworks to obtain a single argumentation framework that reflects the consensus of the group. What would be a good method to achieve this goal? In this paper, we focus on the method arising from social choice theory. An aggregation rule is a function that maps any given profile of attack-relations to a single attack-relation $F : (2^{Arg\times Arg})^n \to 2^{Arg\times Arg}$. We use $N^{\boldsymbol{\rightharpoonup}}_{att} := \{i \in N \mid att \in (\rightharpoonup_i)\}$ to denote the set of *supporters* of the attack *att* in profile $\boldsymbol{\rightharpoonup}$.

Two special families of aggregation rules we consider in this paper are quota rules and dictatorship rules. Both are simple rules that are adapted from other parts of social choice theory, such as judgment aggregation [39] and graph aggregation [35].

Definition 5. *Let $q \in \{1, \ldots, n\}$. The* ***quota rule*** *F_q with quota q accepts all attacks that are supported by at least q agents:*

$$F_q(\boldsymbol{\rightharpoonup}) \quad = \quad \{att \in Arg \times Arg \mid \# N^{\boldsymbol{\rightharpoonup}}_{att} \geqslant q\}$$

The *majority rule* is the quota rule F_q with $q = \lceil \frac{n+1}{2} \rceil$. Two further quota rules are also of special interest. The *unanimity rule* accepts only attacks that are supported by everyone, i.e., this rule is equivalent to F_q with $q = n$. The *nomination rule* is the quota rule F_q with $q = 1$. Despite being a somewhat extreme choice, the nomination rule has some intuitive appeal in the context of argumentation, as it reflects the idea that we should take seriously any conflict between arguments raised by at least one member of the group.

Definition 6. *The* ***dictatorship rule*** *F_{D_i} of dictator $i \in N$ accepts all attacks that are accepted by agent i:*

$$F_{D_i}(\boldsymbol{\rightharpoonup}) \quad = \quad \rightharpoonup_i$$

Thus, under a dictatorship, to compute the outcome, we simply copy the attack-relation of the dictator. Intuitively, dictatorships in particular are unattractive rules, as they unfairly exclude everyone except i from the decision process.

Now, we present several intuitively desirable properties of aggregation rules. Such properties are called axioms in the literature on social choice theory [2]. All of these axioms are adapted from axioms formulated in the literature on graph aggregation [35] and have been defined in the work by Chen and Endriss [23].

Definition 7. *An aggregation rule is said to be* neutral *if $N^{\boldsymbol{\rightharpoonup}}_{att} = N^{\boldsymbol{\rightharpoonup}}_{att'}$ implies that $att \in F(\boldsymbol{\rightharpoonup}) \Leftrightarrow att' \in F(\boldsymbol{\rightharpoonup})$ for all profiles $\boldsymbol{\rightharpoonup}$ and all attacks att and att′.*

Definition 8. *An aggregation rule is said to be* independent *if $N^{\boldsymbol{\rightharpoonup}}_{att} = N^{\boldsymbol{\rightharpoonup}'}_{att}$ implies that $att \in F(\boldsymbol{\rightharpoonup}) \Leftrightarrow att \in F(\boldsymbol{\rightharpoonup}')$ for all attacks att and all profiles $\boldsymbol{\rightharpoonup}$ and $\boldsymbol{\rightharpoonup}'$.*

Definition 9. *An aggregation rule is said to be* unanimous *if $F(\boldsymbol{\rightharpoonup}) \supseteq (\rightharpoonup_1) \cap \cdots \cap (\rightharpoonup_n)$ holds for all profiles $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$.*

Definition 10. *An aggregation rule is said to be* grounded *if $F(\boldsymbol{\rightharpoonup}) \subseteq (\rightharpoonup_1) \cup \cdots \cup (\rightharpoonup_n)$ holds for all profiles $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$.*

Definition 11. *An aggregation rule is said to be* anonymous *if $F(\boldsymbol{\rightharpoonup}) = F(\rightharpoonup_{\pi(1)}, \ldots, \rightharpoonup_{\pi(n)})$ holds for all profiles $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$ and all permutations $\pi : N \to N$.*

Definition 12. *An aggregation rule is said to be* monotonic *if* $N_{att}^{\rightharpoonup} \subseteq N_{att}^{\rightharpoonup'}$ *implies that* $att \in F(\rightharpoonup) \rightarrow att \in F(\rightharpoonup')$ *for all profiles of attack-relations* $\rightharpoonup$ *and* $\rightharpoonup'$ *and all attacks att.*

Thus, an aggregation rule is neutral if two attacks receive the same number of votes in a profile, meaning that their acceptance statuses are the same in the outcome, i.e., attacks are treated symmetrically; an aggregation rule is independent if the acceptance of an attack depends only on its supporters; unanimity assumes that if an attack is accepted by everyone, then it should be accepted in the collective outcome; and groundedness postulates that only attacks with at least one supporter can be collectively accepted. Anonymity is a symmetry requirement regarding agents in the sense that all agents should be treated fairly, while monotonicity postulates that additional support for an accepted attack should never cause it to be rejected.

We consider the preservation of the semantic properties of argumentation frameworks. A semantic property $P \subseteq 2^{Arg \times Arg}$ is the set of all attack-relations on Arg that satisfy P; we denote it by $P(\rightharpoonup)$. For example, nonemptiness of the grounded extension is a simple semantic property, and an AF satisfies this property if there is at least one argument that is not attacked by any argument in AF.

Definition 13 (Preservation)**.** *Given a finite set Arg of arguments and a set of* $N = \{1, \cdots, n\}$ *agents, suppose that each agent provides an argumentation framework that reflects her individual views on the statuses of possible attacks between arguments. An aggregation rule* F *is said to preserve a property* P *if for every profile* $\rightharpoonup$*,* $P(\rightharpoonup_i)$ *holds for all agents* $i \in N$*; then,* $P(F(\rightharpoonup))$*.*

Thus, in the case that all agents' attack-relations satisfy P, F preserves P if the outcome of F satisfies P as well. The semantic properties we are interested in include the following:

- conflict-freeness
- admissibility
- being an extension under a specific semantics
- nonemptiness of the grounded extension
- coherence
- acyclicity

Conflict-freeness is a semantic property that requires that for all sets $\Delta \subseteq Arg$, whenever Δ is conflict-free in $\langle Arg, \rightharpoonup_i \rangle$ for all agents $i \in N$, Δ is also conflict-free in $\langle Arg, F(\rightharpoonup) \rangle$. If this is the case, then we say that F preserves conflict-freeness. *Admissibility* can be defined in the same way. The semantic property of *being an extension under a specific semantics* requires that given a set of arguments Δ, if Δ is an extension of a given semantics in $\langle Arg, \rightharpoonup_i \rangle$ for all agents $i \in N$, then Δ is also an extension of the semantics of $\langle Arg, F(\rightharpoonup) \rangle$. *Nonemptiness of the grounded extension* has been mentioned previously. Finally, *coherence* and *acyclicity* are also attractive properties because, if satisfied by an argumentation framework, they ensure that two or more semantics will coincide and result in the same recommendations about which arguments to accept, thereby making decisions less controversial. Skeptical reasoning with the preferred semantics is coNP-complete instead of Π_2^P-complete for coherent AFs (because of the equivalence with the stable semantics), and all classical semantics are polynomially computable for acyclic AFs (because of the equivalence with the grounded semantics) [31, 18]. Thus, coherence and acyclicity are two desirable properties for practical reasons.

It is worth noting that the topological properties mentioned in Section 2 can also be considered a special subset of semantic properties. For example, let us consider acyclicity, which can be considered a semantic property to be preserved; it is a natural topological property of argumentation frameworks. If it is imposed as a topological condition on the domains of individual AFs, as we will see in the following sections, then we call it a topological restriction.

4 Topological restrictions: conditions on individual argumentation frameworks

In this section, we introduce the notion of topological restriction for the aggregation of the attack-relations of argumentation frameworks. What are the intuitions behind this notion? First, while it is easy to verify that most semantic properties cannot be preserved by the majority rule[1], it is generally not possible to construct an aggregation rule that satisfies all desirable axiomatic requirements. As an example, we present the following impossibility theorem.

Theorem 1 (Chen and Endriss, 2019)**.** *For $|Arg| \geqslant 5$, any unanimous, grounded, and independent aggregation rule F that preserves either complete or preferred extensions must be a dictatorship.*

To prove Theorem 1, Chen and Endriss use a technique developed by Endriss and Grandi for the more general framework of graph aggregation, which in turn was inspired by the seminal work on preference aggregation of Arrow [1]. Clearly, Theorem 1 is an impossibility result. At the heart of Theorem 1 (as well as other impossibility results) lie three types of conditions: the axioms of aggregation rules, the semantic properties of argumentation frameworks, and the argumentation frameworks that are allowed to be input. To cope with such undesirable results, one possible approach is to restrict the domain of the allowed sets of argumentation frameworks.

Before going any further, we recall various domain restrictions that aim to address impossibility results in the literature on social choice theory. Introduced by Black [10], single-peakedness is one of the most widely studied domain restrictions for preference aggregation. A profile of individual preferences is single-peaked if every individual preference decreases from a most preferred alternative in any direction. Introduced by List [42], *unidimensional alignment* is a known method of domain restriction for judgment aggregation. The idea of unidimensional alignment is that only profiles of individual sets of judgments satisfying a certain structural condition are admissible. *Value restriction* is another type of domain restriction, the idea of which was first introduced by Sen [45] for preference aggregation and later generalized by Dietrich and List [28] for judgment aggregation. They show that if a profile is *value-restricted* in the sense that for every minimal inconsistent subset X of the agenda, there exist two formulas $\varphi, \psi \in X$ such that no agent accepts both φ and ψ, then the outcome of the majority rule will be consistent.

Recently, Chen has considered value restriction during the aggregation of extensions of AFs [20]. He assumes that individual agents choose different extensions when confronted with the same abstract argumentation framework and studies the preservation of the properties of these extensions. Chen

[1] A notable exception is conflict-freeness, which can be preserved by the majority rule [23].

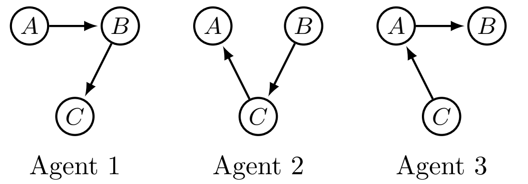

Figure 2: Example for a profile with $Arg = \{A, B, C, D\}$.

uses a formula Γ to describe such a property of extensions and refers to Γ as an integrity constraint. He shows that if for every prime implicate π of the integrity constraint Γ of a given semantic property there exist two distinct literals such that no agent rejects both, then the majority rule preserves admissible outcomes [20].

We propose restricting the inputs to an aggregation rule such that only argumentation frameworks with a specific feature are allowed to be acted on by the aggregation rule. In the work by Chen and Endriss, whose model we adopt in this paper, no restriction is imposed on the argumentation frameworks put forward by individual agents. While there are many argumentation frameworks that contain undesirable features, it is natural to restrict the aggregation inputs to the family of argumentation frameworks without such features.

Definition 14. *A profile* $\rightharpoonup\!\!\!\!\rightharpoonup = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$ *is* topologically restricted *with respect to a constraint* Γ *if and only if* $\rightharpoonup_i$ *satisfies* Γ *for all* $i \in N$.

Thus, given a constraint Γ that is a topological property of argumentation frameworks, a profile is topologically restricted with respect to Γ if every individual argumentation framework satisfies Γ. When we perform aggregation on such a profile, only argumentation frameworks satisfying Γ are allowed to be acted on by the aggregation rules. While most preservation results for demanding properties are negative [23], possible results may be obtained when suitable restrictions are imposed. Consider the following example:

Example 2. Let us consider an example that illustrates the preservation of the nonemptiness of the grounded extension with the majority rule. Recall that the majority rule includes an attack if and only if a majority of the individual agents support it. Consider three agents, of whom the first supports $A \rightharpoonup B$ and $B \rightharpoonup C$, the second supports $B \rightharpoonup C$ and $C \rightharpoonup A$, and the third supports $C \rightharpoonup A$ and $A \rightharpoonup B$. This profile is illustrated in Figure 2. Clearly, every individual argumentation framework in this profile satisfies nonemptiness of the grounded extension, as at least one argument is not attacked by any other argument. If we apply the majority rule to this profile, then we obtain an argumentation framework that contains three attacks, $A \rightharpoonup B$, $B \rightharpoonup C$, and $C \rightharpoonup A$, which form a cycle, violating the property of nonemptiness of the grounded extension. However, if the second agent supports only $B \rightharpoonup C$ and the third agent supports only $A \rightharpoonup B$, i.e., for every individual AF, there is at least one argument not attacked by any other argument, then nonemptiness of the grounded extension will be preserved, as only $A \rightharpoonup B$ and $B \rightharpoonup C$ are accepted by the majority rule and A is not attacked by any other argument. Thus, we can consider requiring that at least one argument is not attacked by any argument as a topological restriction Γ. If a profile is topologically restricted with respect to Γ, then the majority rule will preserve the nonemptiness of the grounded extension. $\triangle$

Example 2 concerns only a specific profile. We now present a proposition that is more concrete, showing that if a profile is topologically restricted with respect to a constraint Γ, then every plausible aggregation rule preserves a certain semantic property, namely, the nonemptiness of the grounded extension. Here, we recall the work by Chen and Endriss, who present a preservation result for the nonemptiness of the grounded extension.

Theorem 2 (Chen and Endriss, 2019)**.** *If $|Arg| \geqslant n$, then under any neutral and independent aggregation rule F that preserves the nonemptiness of the grounded extension, at least one agent must have veto powers.*

Proposition 3. *Let Γ be a topological property that requires the existence of an argument $A \in Arg$ that is unattacked in $\rightharpoonup_i$ for all $i \in N$. Given a profile $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$ that is topologically restricted with respect to Γ, every aggregation rule that is grounded preserves the nonemptiness of the grounded extension.*

Proof. Let F be a grounded aggregation rule. Consider a profile of attack-relations $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \cdots, \rightharpoonup_n$). Suppose that $A \in Arg$ is an unattacked argument in $\rightharpoonup_i$ for all $i \in N$. Clearly, because F is grounded, i.e., $F(\boldsymbol{\rightharpoonup}) \subseteq \rightharpoonup_1 \cup, \cdots, \cup \rightharpoonup_n$, no argument attacks A in $F(\boldsymbol{\rightharpoonup})$. □ □

Thus, a positive result is obtained when the profile is topologically restricted with respect to a topological property that is weak and easy to satisfy. Proposition 3 provides a clue on how to overcome negative results during aggregation. In the following section, we study additional topological restrictions, including restrictions based on notable topological properties in the literature, such as acyclicity, symmetry, coherence, and determinedness, as well as t-self-defense, t-attack, and t-acy, three topological restrictions that generalize classical social-choice-theoretic domain conditions, and we show that the majority rule behaves well with these restrictions.

5 Topological restrictions based on topological properties

In this section, we study several topological restrictions that are based on well-known topological properties, including acyclicity, symmetry, coherence, and determinedness. All of them have been studied in the literature on formal argumentation. We present several results on the preservation of properties under topological restrictions. Most of our results have the following form: there exists an aggregation rule preserving one semantic property that coincides with a second semantic property, i.e., the two semantics are in agreement; if the members of a profile of argumentation frameworks satisfy a certain topological restriction, then the preservation result for one semantics can be extended to the other.

5.1 Acyclicity

Acyclicity is an important topological property of argumentation frameworks. As mentioned in previous sections, if an argumentation framework is acyclic, then it contains a single extension that is the only complete, preferred and stable extension. When this topological property is imposed as a condition on individual argumentation frameworks to guarantee the preservation of semantic properties, we call it a topological restriction.

Definition 15. *A profile* $\rightharpoonup = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$ *is topologically restricted with respect to acyclicity if* $\rightharpoonup_i$ *is acyclic for all* $i \in N$.

Fact 4. *In the case of a finite argumentation framework, well-foundedness coincides with acyclicity of the attack-relations.*

Theorem 5 (Dung, 1995)**.** *Every acyclic argumentation framework has exactly one complete extension that is grounded, preferred and stable.*

Proposition 6 (Chen and Endriss, 2019)**.** *The nomination rule preserves stable extensions.*

Fact 7. *Every stable extension is preferred and complete.*

We now present the preservation results for preferred and complete extensions under the topological restriction of acyclicity. The preservation of both semantic properties is discussed in depth by Chen and Endriss in [23], who show that the preservation of extensions of either a preferred or complete semantics is impossible by means of a "simple" aggregation rule (a rule that satisfies three "fair" axioms), unless the rule in use is a dictatorship.

Theorem 8 (Chen and Endriss, 2019)**.** *For* $|Arg| \geqslant 5$*, any unanimous, grounded, and independent aggregation rule* F *that preserves either preferred or complete extensions must be a dictatorship.*

Theorem 9. *For any profile of attack-relations* $\rightharpoonup = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$*, if* $\rightharpoonup$ *is topologically restricted with respect to acyclicity, then the nomination rule preserves grounded, complete, and preferred extensions.*

Proof. Let F be the nomination rule. Suppose that $\Delta \subseteq Arg$ is a set of arguments that is grounded, complete, or preferred in $\rightharpoonup_i$ for all $i \in N$. According to Theorem 5, Δ is stable in $\rightharpoonup_i$ for all $i \in N$. Thus, because F preserves stable extensions, Δ is stable in $F(\rightharpoonup)$. Because every stable extension is grounded, complete, and preferred, we get that Δ is grounded, complete, and preferred in $F(\rightharpoonup)$. The proof is complete. $\square$ $\square$

5.2 Symmetry

In this section, we consider a topological restriction based on the topological property of symmetry. Before going any further, we present a result regarding the preservation of conflict-freeness in [23], which shows that every plausible aggregation rule preserves it.

Theorem 10 (Chen and Endriss, 2019)**.** *Every aggregation rule* F *that is grounded preserves conflict-freeness.*

We also present a result concerning the relation between admissibility and conflict-freeness presented in [25], which shows that admissible sets and conflict-free sets coincide in symmetric argumentation frameworks.

Proposition 11 (Coste-Marquis et al., 2005)**.** *Let* $AF = \langle Arg, \rightharpoonup \rangle$ *be a symmetric argumentation framework; then, a set of arguments* $\Delta \in Arg$ *is admissible if and only if it is conflict-free.*

Definition 16. *A profile* $\rightharpoonup = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$ *is topologically restricted with respect to symmetry if* $\rightharpoonup_i$ *is symmetric for all* $i \in N$.

With Theorem 10 and Proposition 11, we are ready to present a preservation result for admissibility under the topological restriction of symmetry.

Theorem 12. *For any profile of attack-relations* $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$*, if* $\boldsymbol{\rightharpoonup}$ *is topologically restricted with respect to symmetry, then every aggregation rule that is grounded and neutral preserves admissibility.*

Proof. Consider a profile of attack-relations $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$. Let F be an aggregation rule that is grounded and neutral. Let $\Delta \subseteq Arg$ be a set of arguments that is admissible in $\rightharpoonup_i$ for all $i \in N$. Clearly, Δ is conflict-free in $\rightharpoonup_i$ for all $i \in N$. Because F preserves conflict-freeness (cf. Theorem 10), we get that Δ is conflict-free in $F(\boldsymbol{\rightharpoonup})$. According to the neutrality of F and the fact that the profile is topologically restricted with respect to symmetry, for every pair of arguments $A, B \in Arg$, $A \rightharpoonup B$ and $B \rightharpoonup A$ are treated symmetrically, and they receive the same number of votes, i.e., if $A \rightharpoonup B$ is accepted by F, so is $B \rightharpoonup A$. Thus, $F(\boldsymbol{\rightharpoonup})$ is symmetric. Combining this result with Proposition 11, we obtain that Δ is admissible in $F(\boldsymbol{\rightharpoonup})$. The proof is complete. □ □

Proposition 13 (Coste-Marquis et al., 2005)**.** *Every symmetric argumentation framework is coherent.*

Recall that coherence is a semantic property ensuring that the stable and preferred semantics coincide. We say that an aggregation rule F preserves coherence if whenever $\langle Arg, \rightharpoonup_i \rangle$ is coherent for all $i \in N$, $F(\boldsymbol{\rightharpoonup})$ is also coherent. Chen and Endriss [23] show that preservation of coherence is impossible unless we use dictatorships.

Theorem 14 (Chen and Endriss, 2019)**.** *For* $|Arg| \geqslant 4$*, any unanimous, grounded, and independent aggregation rule* F *that preserves coherence must be a dictatorship.*

However, when the profile under consideration is topologically restricted with respect to symmetry, impossibility results can be avoided.

Theorem 15. *For any profile of attack-relations* $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$*, if* $\boldsymbol{\rightharpoonup}$ *is topologically restricted with respect to symmetry, then any aggregation rule that is grounded and neutral preserves coherence.*

Proof. Let F be an aggregation rule that is grounded and neutral. Consider a pair of arguments $A, B \in Arg$ as well as the attacks $A \rightharpoonup B$ and $B \rightharpoonup A$ between them. According to the fact that $\boldsymbol{\rightharpoonup}$ is a profile that is topologically restricted with respect to symmetry and the fact that F is an aggregation rule that is grounded and neutral, we find that $A \rightharpoonup B$ and $B \rightharpoonup A$ receive the same number of votes and are treated symmetrically by F. Thus, if $A \rightharpoonup B$ is accepted, then $B \rightharpoonup A$ is accepted as well. Therefore, $F(\boldsymbol{\rightharpoonup})$ is a symmetric argumentation framework. Combining this finding with Proposition 13, we get that $F(\boldsymbol{\rightharpoonup})$ is coherent. □ □

Recall that Theorem 1 has shown that only dictatorships preserve preferred extensions. Interestingly, when the topological restriction of symmetry is imposed on individual argumentation frameworks, we obtain a much more positive result.

Theorem 16. *For any profile of attack-relations* $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$*, if* $\boldsymbol{\rightharpoonup}$ *is topologically restricted with respect to symmetry, then the nomination rule preserves preferred extensions.*

Proof. Let F be the nomination rule. Suppose that $\Delta \subseteq Arg$ is a set of arguments that is preferred in $\rightharpoonup_i$ for all $i \in N$. According to Proposition 13, Δ is stable in $\rightharpoonup_i$ for all $i \in N$. Thus, because F preserves stable extensions (cf. Proposition 6), Δ is stable in $F(\rightharpoonup)$. Because every stable extension is preferred, Δ is preferred in $F(\rightharpoonup)$. □ □

5.3 Coherence and determinedness

In this section, we consider topological restrictions based on the topological properties of coherence and determinedness.

Definition 17. *A profile $\rightharpoonup = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$ is topologically restricted with respect to coherence if $\rightharpoonup_i$ is coherent for all $i \in N$.*

The following fact can be obtained from the definition of *coherence* of argumentation frameworks.

Fact 17. *Given an argumentation framework AF that is coherent, each preferred extension of AF is stable.*

As we have seen, the nomination rule preserves stable extensions on unrestricted profiles of argumentation frameworks. We can obtain that on coherent profiles, this rule also preserves preferred extensions.

Theorem 18. *If $\rightharpoonup$ is a profile that is topologically restricted with respect to coherence, then the nomination rule preserves preferred extensions.*

Proof. Let F be the nomination rule. Suppose that $\Delta \subseteq Arg$ is a set of arguments that is preferred in $\rightharpoonup_i$ for all $i \in N$. According to Fact 17, Δ is stable in $\rightharpoonup_i$ for all $i \in N$. Thus, because F preserves stable extensions (cf. Proposition 6), Δ is stable in $F(\rightharpoonup)$. Because every stable extension is preferred, Δ is preferred in $F(\rightharpoonup)$. The proof is complete. □ □

Let us briefly state the relation between the topological restrictions of acyclicity and coherence. Every acyclic argumentation framework is coherent, and this relation can be extended to profiles satisfying topological restrictions based on the two properties.

Fact 19. *Every profile that is topologically restricted with respect to acyclicity is also topologically restricted with respect to coherence.*

We omit the relatively simple proof of Fact 19. Thus, Theorem 18 can be extended to profiles that are topologically restricted with respect to acyclicity, as expected from Theorem 9. However, Fact 19 does not hold in the opposite direction; therefore, we are not able to extend the result for grounded or complete extensions in Theorem 9 to profiles that are topologically restricted with respect to coherence.

Now, let us consider the restriction based on determinedness.

Definition 18. *A profile $\rightharpoonup = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$ is topologically restricted with respect to determinedness if $\rightharpoonup_i$ is determined for all $i \in N$.*

Fact 20. *Given an argumentation framework AF that is determined, the grounded extension of AF is stable.*

Theorem 21. *If $\rightharpoonup$ is a profile that is topologically restricted with respect to determinedness, then the nomination rule preserves the grounded extension.*

Proof. Let F be the nomination rule. Suppose that $\Delta \subseteq Arg$ is a set of arguments that is the grounded extension of $\langle Arg, \rightharpoonup_i \rangle$ for all $i \in N$. According to Fact 20, Δ is stable in $\rightharpoonup_i$ for all $i \in N$. Thus, because F preserves stable extensions (cf. Proposition 6), Δ is stable in $F(\rightharpoonup)$. Because every stable extension is the grounded extension, we obtain that Δ is the grounded extension of $F(\rightharpoonup)$. $\square$ $\square$

It is worth mentioning the relation between the topological restrictions of acyclicity and determinedness.

Fact 22. *Every profile that is topologically restricted with respect to acyclicity is also topologically restricted with respect to determinedness.*

The proof of Fact 22 is simple, and we omit it here. Thus, we can extend Theorem 21 to profiles that are topologically restricted with respect to acyclicity, as shown in Theorem 9. However, Fact 22 does not hold in the opposite directions; therefore, we are not able to extend the result for complete or preferred extensions in Theorem 9 to profiles that are topologically restricted with respect to determinedness.

6 Majority voting with topological restrictions

In this section, we focus on majority voting with topological restrictions. We define three topological restrictions called t-self-defense, t-attack, and t-acy, which generalize classical social-choice-theoretic domain conditions. For every topological restriction, we show that if a profile of attack-relations is topologically restricted with respect to it, then the majority rule preserves a certain semantic property during aggregation.

Before introducing our new topological restrictions, we present the notion of *the union of attack-relations* of a profile of attack-relations.

Definition 19. *Given a profile $\rightharpoonup = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$, we denote the union of attack-relations of $\rightharpoonup$ by $\rightharpoonup_u$, i.e., $\rightharpoonup_u = \rightharpoonup_i \cup \cdots \cup \rightharpoonup_n$.*

In other words, the union of attack-relations of a profile is the attack-relation that includes those attacks accepted by at least one agent. For instance, in Example 3, $\rightharpoonup_u = \{A_1 \rightharpoonup B, A_2 \rightharpoonup B, A_3 \rightharpoonup B, B \rightharpoonup C\}$.

We recall an extension of Dung's framework [32] in which a weight on each attack is added: the resulting *weighted argumentation framework* is a triple consisting of a set of arguments, an attack-relation between them, and a function assigning a natural number to each attack. The union of attack-relations can be regarded as a weighted argumentation framework in which the weight of an attack is assigned a value of 1 if there is any agent supporting this attack. Thus, our notion provides an interpretation of the weights of attacks, forging a link between argumentation theory and social choice theory.

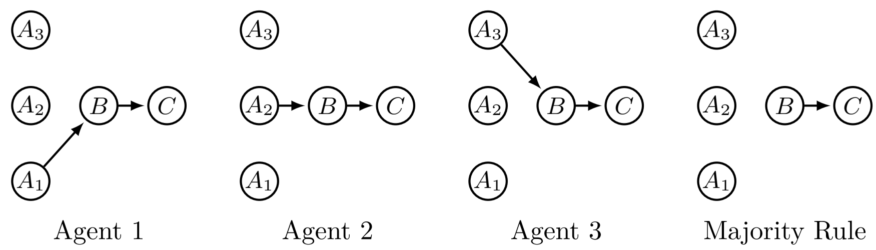

Figure 3: Scenarios used in Example 3.

6.1 Topological condition for preserving admissibility

Admissibility is a property at the heart of all classical semantics. We first present an example showing that the majority rule does not preserve admissibility with unrestricted inputs.

Example 3. Consider the profile illustrated in Figure 3, where $\{A_1, A_2, A_3, C\}$ is admissible in each individual's argumentation framework but is not admissible in the outcome of the majority rule. Thus, the majority rule does not preserve admissibility. $\triangle$

Definition 20. *Given a profile of attack-relations* $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$*, we say that* $\boldsymbol{\rightharpoonup}$ *is topologically restricted with respect to* t-self-defense *if for every attack* $B \rightharpoonup C \in \rightharpoonup_u$ *whose attacker* B *has two or more attackers in* $\langle Arg, \rightharpoonup_u \rangle$*, for every pair of attackers* A_i *and* A_j *of* B*, no agent rejects both* $A_i \rightharpoonup B$ *and* $A_j \rightharpoonup B$*.*

In other words, for every attack $att = B \rightharpoonup C \in \rightharpoonup_u$, we denote the attackers of B by $A_1, \cdots, A_k$ with $k \geqslant 2$, i.e., $A_1 \rightharpoonup B, \cdots, A_k \rightharpoonup B \in \rightharpoonup_u$. Then, for every pair of attackers A_i and A_j of B, no agent rejects both $A_i \rightharpoonup B$ and $A_j \rightharpoonup B$ in a profile that is topologically restricted with respect to t-self-defense.

For example, the profile in Example 3 is not topologically restricted with respect to t-self-defense. To see this, let us consider $B \rightharpoonup C$. The attackers of B in $\langle Arg, \rightharpoonup_u \rangle$ are A_1, A_2, and A_3; for the pair of attacks $A_1 \rightharpoonup B$ and $A_2 \rightharpoonup B$, agent 3 does not support either of them.

Informally speaking, t-self-defense reacts to a particular kind of agreement among agents: for every attack and every pair of counter-attacks that defeats such attack, no individual rejects both. For some properties, what we are truly interested in is the preservation of entire collections of properties. For example, for every set of arguments Δ, we may wish the admissibility of Δ to be preserved under aggregation. Similar to the previous value-restriction conditions for judgment aggregation [28], the new condition is sufficient for the preservation of admissibility for any set of arguments. Before going any further, we recall a result concerning the preservation of admissibility with unrestricted inputs.

Theorem 23 (Chen and Endriss, 2019)**.** *For* $|Arg| \geqslant 4$*, the only unanimous, grounded, anonymous, independent, and monotonic aggregation rule* F *that preserves admissibility is the nomination rule.*

As a consequence of Theorem 23, the majority rule, an aggregation rule that satisfies all five axioms mentioned, does not preserve admissibility. However, if a profile is topologically restricted with respect to t-self-defense, then admissibility is preserved in the majority outcome.

Theorem 24. *If the number of agents is odd, then for any profile of attack-relations $\rightharpoonup = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$, if $\rightharpoonup$ is topologically restricted with respect to t-self-defense, then the majority rule preserves admissibility.*

Proof. Assume that $\Delta \subseteq Arg$ is admissible in $\rightharpoonup_i$ for all $i \in N$. Let F be the majority rule. According to Theorem 10, Δ is conflict-free in $F(\rightharpoonup)$. It remains to be shown that Δ is self-defending in $F(\rightharpoonup)$. To achieve this goal, we need to show that for every argument $C \in \Delta$, if C is attacked by some argument B, then B is attacked by some argument in Δ in the outcome of the majority rule.

Suppose that $B \rightharpoonup C \in F(\rightharpoonup)$; then, $B \rightharpoonup C \in \rightharpoonup_u$. If B does not have any attacker, then C is not defended by Δ in every AF that supports $B \rightharpoonup C$. Consequently, Δ is not admissible in such AFs, contradicting our assumption. If B has only one attacker in $\langle Arg, \rightharpoonup_u \rangle$, which we denote by A, then any agent who supports $B \rightharpoonup C$ would also be required to support $A \rightharpoonup B$, meaning that the majority of agents support $A \rightharpoonup B$. Thus, in this scenario, $B \rightharpoonup C$ and $A \rightharpoonup B$ receive the same number of votes, which also represents a majority of support from agents. If $A \notin \Delta$, then Δ is not self-defending in such agents' argumentation frameworks, contradicting our earlier assumption. Thus, $A \in \Delta$, meaning that C is defended by Δ.

If B has two or more attackers in $\langle Arg, \rightharpoonup_u \rangle$, we denote the attackers of B by $A_1, \cdots, A_k$. According to the assumption that $\rightharpoonup$ is topologically restricted with respect to t-self-defense, for every pair of attackers A_i and A_j of B, no agent rejects both $A_i \rightharpoonup B$ and $A_j \rightharpoonup B$. We now show that C is defended by Δ in $F(\rightharpoonup)$. If there are two or more arguments in $A_1, \cdots, A_k$ that are included in Δ, we take two of them and denote them by A_i and A_j. Clearly, one of $A_i \rightharpoonup B$ and $A_j \rightharpoonup B$ is supported by the majority of agents. Thus, A is defended by Δ in $F(\rightharpoonup)$. If there is only one argument in $A_1, \cdots, A_k$ that is included in Δ, we denote it by A_i. Clearly, $A_i \rightharpoonup B$ is supported by agents who support $B \rightharpoonup C$, i.e., $A_i \rightharpoonup B$ is accepted by F, meaning that C is defended by Δ in $F(\rightharpoonup)$ as well. Regarding the scenario in which no argument in $A_1, \cdots, A_k$ is included in Δ, we note that this is impossible because for agents who support $B \rightharpoonup C$, Δ is not self-defending in their individual argumentation frameworks. □ □

Let us return to Example 3, the union of attack-relations of the profile $\rightharpoonup_u = \{A_1 \rightharpoonup B, A_2 \rightharpoonup B, A_3 \rightharpoonup B, B \rightharpoonup C\}$. Clearly, this profile is not topologically restricted with respect to t-self-defense because for $B \rightharpoonup C$, the attacker B in turn has three attackers, and for every pair of attackers of B in $\langle Arg, \rightharpoonup_u \rangle$, there is at least one agent who rejects both.

Example 4. Now, we consider the profile illustrated in Figure 4, in which $\rightharpoonup_1 = \{A_1 \rightharpoonup B, A_2 \rightharpoonup B\}$, $\rightharpoonup_2 = \{A_2 \rightharpoonup B, A_3 \rightharpoonup B\}$, and $\rightharpoonup_3 = \{A_3 \rightharpoonup B, A_1 \rightharpoonup B\}$, and we wish to know whether $\{A_1, A_2, A_3, C\}$ is admissible in the outcome of the majority rule. Clearly, the profile is topologically restricted with respect to t-self-defense. We can see that the union of attack-relations of the profile is $\rightharpoonup_u = \{A_1 \rightharpoonup B, A_2 \rightharpoonup B, A_3 \rightharpoonup B, B \rightharpoonup C\}$. Here, $B \rightharpoonup C$ is the only attack for which the attacker is, in turn, attacked. For every pair of attackers of B, no agent rejects both attacks against B; for example, for A_1 and A_2, no agent rejects both $A_1 \rightharpoonup B$ and $A_2 \rightharpoonup B$. $\{A_1, A_2, A_3, C\}$ is admissible in every individual agent's argumentation framework, and it is also admissible in the outcome of the majority rule, as expected. △

Recall that Theorem 24 applies only when the number of agents is odd. The reader might still wonder whether the theorem could maybe be strengthened to the cases where the number of agents

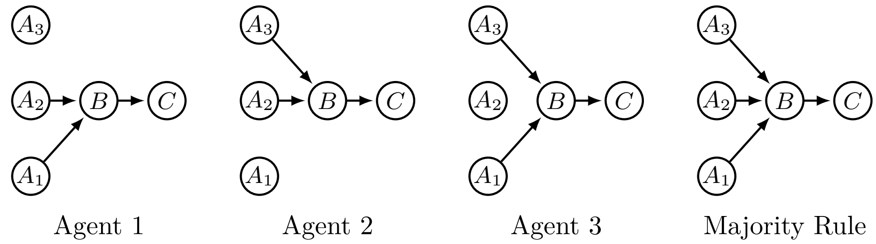

Figure 4: Scenarios used in Example 4.

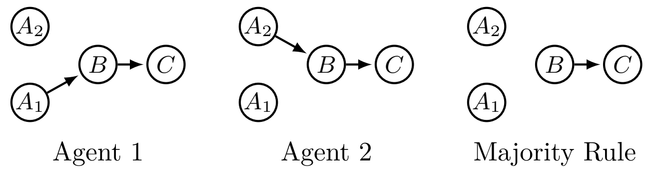

Figure 5: A counterexample.

is even. There are obvious counterexamples. For example, we consider a profile with two agents, which is illustrated in Figure 5, in which $\rightharpoonup_1 = \{A_1 \rightharpoonup B, B \rightharpoonup C\}, \rightharpoonup_2 = \{A_2 \rightharpoonup B, B \rightharpoonup C\}$, and we want to know whether $\Delta = \{A_1, A_2, C\}$ is admissible in the outcome of the majority rule (it is clearly admissble in every individual AF). It is easy to verify that the profile is topologically restricted with respect to t-self-defense. While $\{A_1, A_2, C\}$ is admissible in every individual AF, it is not admissible in the outcome of the majority rule.

6.2 Topological condition for preserving stable extensions

We now consider the topological condition for the preservation of stable extensions with the majority rule.

Definition 21. *Given a profile of attack-relations* $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$*, we say that* $\boldsymbol{\rightharpoonup}$ *is topologically restricted with respect to* t-attack *if for any given argument* $C \in Arg$ *with two or more attackers in* $\langle Arg, \rightharpoonup_u \rangle$*, for every pair of attackers* B_i *and* B_j *of* C*, no agent rejects both* $B_i \rightharpoonup C$ *and* $B_j \rightharpoonup C$*.*

In other words, given a profile $\boldsymbol{\rightharpoonup}$ that is topologically restricted with respect to t-attack, if C has no attacker in $\langle Arg, \rightharpoonup_u \rangle$, then no argument in $\langle Arg, \rightharpoonup_i \rangle$ attacks C for every $i \in N$; if C has only one attacker in $\langle Arg, \rightharpoonup_u \rangle$, which we denote by B, then at least one agent supports $B \rightharpoonup C$ and no other argument attacks C in all agents' argumentation frameworks; and if C has two or more attackers in $\langle Arg, \rightharpoonup_u \rangle$, which we denote by $B_1, \cdots, B_k$, then $B_i \rightharpoonup C$ is supported by one or more agents for $1 \leqslant i \leqslant k$ (every attack is supported by one or more agents). Informally, t-attack reacts to a particular kind of agreement among agents: for every pair of attacks that defeat the same argument, no individual rejects both.

Theorem 25. *If the number of agents is odd, then for any profile of attack-relations* $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$*, if* $\boldsymbol{\rightharpoonup}$ *is a profile that is topologically restricted with respect to t-attack, then the majority rule preserves stable extensions.*

Proof. Assume that $\Delta \subseteq Arg$ is stable in $\rightharpoonup_i$ for all $i \in N$. Let F be the majority rule. According to Theorem 10, Δ is conflict-free in $F(\boldsymbol{\rightharpoonup})$. It remains to be shown that every argument $C \in Arg \backslash \Delta$ is attacked by Δ in $F(\boldsymbol{\rightharpoonup})$.

If C does not have any attacker in $\langle Arg, \rightharpoonup_u\rangle$, then C does not have any attacker in $\langle Arg, \rightharpoonup_i\rangle$ for all $i \in N$, which implies that C is contained in every stable extension of every individual AF, contradicting our assumption. If C has only one attacker in $\langle Arg, \rightharpoonup_u\rangle$, we denote it by B. Clearly, $B \in \Delta$. Then, for every agent, C is attacked by B in her individual argumentation framework, i.e., $B \rightharpoonup C \in \rightharpoonup_i$ for all $i \in N$, meaning that C is attacked by Δ in $F(\boldsymbol{\rightharpoonup})$.

If C has two or more attackers in $\langle Arg, \rightharpoonup_u\rangle$, we denote such attackers by $B_1, \cdots, B_k$. According to the assumption that $\boldsymbol{\rightharpoonup}$ is topologically restricted with respect to t-attack, for every pair of attackers B_i and B_j of C, no agent rejects both $B_i \rightharpoonup C$ and $B_j \rightharpoonup C$. We now show that C is attacked by Δ in $F(\boldsymbol{\rightharpoonup})$. If there are two or more arguments in $B_1, \cdots, B_k$ that are included in Δ, we take two of them and denote them by B_i and B_j. Clearly, because no agent rejects both $B_i \rightharpoonup C$ and $B_j \rightharpoonup C$, one of $B_i \rightharpoonup C$ and $B_j \rightharpoonup C$ is supported by the majority of agents. Thus, C is attacked by Δ in $F(\boldsymbol{\rightharpoonup})$. If there is only one argument in $B_1, \cdots, B_k$ that is included in Δ, we denote it by B_i. Clearly, $B_i \rightharpoonup C$ is supported by every agent, i.e., $B_i \rightharpoonup C$ is accepted by F, meaning that C is attacked by Δ in $F(\boldsymbol{\rightharpoonup})$ as well. Regarding the scenario in which no argument in $B_1, \cdots, B_k$ is included in Δ, we note that this is impossible because Δ is not stable in the agents' individual argumentation frameworks. □ □

Similar to Theorem 24, Theorem 25 assumes that the number of agents n is odd, so we can avoid having to consider tied majorities.

6.3 Topological condition for preserving acyclicity

We have seen that although the majority rule does not preserve either admissibility or stable extensions with unrestricted inputs, properly restricting the domain of permissible profiles can guarantee the preservation of these properties. We continue our investigation by considering the preservation of the semantic property of acyclicity using the same approach.

Definition 22. *Given a profile of attack-relations $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$, we say that $\boldsymbol{\rightharpoonup}$ is topologically restricted with respect to* t-acy *if for any cycle $att_1, \cdots, att_k$ in $\langle Arg, \rightharpoonup_u\rangle$, there is a pair of attacks att_i and att_j such that no agent supports both of them.*

In other words, for every cycle in $\langle Arg, \rightharpoonup_u\rangle$, there is a pair of attacks from the cycle such that every agent rejects at least one of them (rejecting both is admissible; supporting both is not allowed).

Example 5. Given $Arg = \{A, B, C\}$ and a profile $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \rightharpoonup_2, \rightharpoonup_3)$ with $\rightharpoonup_1 = \{A \rightharpoonup B, B \rightharpoonup C\}$, $\rightharpoonup_2 = \{B \rightharpoonup C, C \rightharpoonup A\}$, and $\rightharpoonup_3 = \{A \rightharpoonup B, C \rightharpoonup A\}$, the union of attack-relations is $\rightharpoonup_u = \{A \rightharpoonup B, B \rightharpoonup C, C \rightharpoonup A\}$. There is a single cycle in $\langle Arg, \rightharpoonup_u\rangle$. Clearly, for every pair of attacks from the only cycle in $\langle Arg, \rightharpoonup_u\rangle$, there is at least one agent that supports both, i.e., $\boldsymbol{\rightharpoonup}$ is not topologically restricted with respect to t-acy. Now, let us consider another profile $\boldsymbol{\rightharpoonup}' = (\rightharpoonup'_1, \rightharpoonup'_2, \rightharpoonup'_3)$, in which agents 1 and 2 propose the same AFs as in $\boldsymbol{\rightharpoonup}$, i.e., $\rightharpoonup_1 = \rightharpoonup'_1$ and $\rightharpoonup_2 = \rightharpoonup'_2$, whereas $\rightharpoonup'_3 = A \rightharpoonup B$, i.e., the only difference between the two profiles is that in $\langle Arg, \rightharpoonup'_3\rangle$, agent 3 no longer supports $C \rightharpoonup A$. Clearly, there is a pair of attacks from the only cycle in $\langle Arg, \rightharpoonup'_u\rangle$, namely, $A \rightharpoonup B$ and $C \rightharpoonup A$, for which no agent supports both, i.e., $\boldsymbol{\rightharpoonup}'$ is topologically restricted with respect to t-acy. △

Theorem 26. *For any profile of attack-relations $\rightharpoonup = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$, if $\rightharpoonup$ is a profile that is topologically restricted with respect to t-acy, then the majority rule preserves acyclicity.*

Proof. Let F be the majority rule. Let $\rightharpoonup = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$ be a profile of attack-relations such that att_i is acyclic for all $i \in N$ and $\rightharpoonup$ is topologically restricted with respect to t-acy.

For the sake of contradiction, we assume that $F(\rightharpoonup)$ violates acyclicity, i.e., that there is a cycle in $F(\rightharpoonup)$, and we denote the attacks in this cycle by $att_1, \cdots, att_k$. Every attack in the cycle is supported by at least one agent. According to the assumption that $\rightharpoonup$ is topologically restricted with respect to t-acy and the fact that $att_1, \cdots, att_k$ is a cycle in $\langle Arg, \rightharpoonup_u\rangle$, we get that there is a pair of attacks att_i and att_j (belonging to the cycle) such that no agent supports both of them.

Because both att_i and att_j are accepted by the majority rule, at least one agent supports both att_i and att_j, clearly violating our assumption that $\rightharpoonup$ is topologically restricted with respect to t-acy.
$\square$ $\square$

Recall that in Section 5.1, acyclicity is a topological restriction that is assumed to be satisfied by all individual argumentation frameworks; when it is a feature of argumentation frameworks that must be preserved, i.e., a property that is assumed to be satisfied by all of the individual AFs and the collective AF, it can also be considered a semantic property.

Let us return to Example 5. Clearly, every individual agent's AF in the profile $\rightharpoonup$ satisfies acyclicity. However, the outcome obtained from the majority rule contains $A \rightharpoonup B$, $B \rightharpoonup C$, and $C \rightharpoonup A$, violating acyclicity. As previously mentioned in the example, if agent 3 replaces her attack-relation with $A \rightharpoonup B$, we find that the profile $\rightharpoonup'$ is topologically restricted with respect to t-acy. According to Theorem 26, we obtain a collective AF in which no cycle exists, i.e., acyclicity is preserved.

7 Comparing value restriction and topological restrictions

In this section, we examine the relation between value restriction in judgment aggregation and the topological restrictions discussed in this paper. The topological restrictions we focus on in this section are t-self-defense, t-attack and t-acy, three restrictions that generalize domain conditions for judgment aggregation.

In judgment aggregation, we consider a group of individuals $N = \{1, \cdots, n\}$ making judgments on some propositions. The *agenda*, X, is the set of propositions on which judgments are to be made. The *judgment set* of an individual is the set $A \subseteq X$ of propositions in the agenda that she accepts. A judgment set is *consistent* if it is a logically consistent set of propositions. The profile is a vector of judgment sets $(A_1, \cdots, A_n)$, one for each individual.

Note that a judgment set $A \subseteq X$ is inconsistent if and only if it has a minimal inconsistent subset $Y \subseteq X$, i.e., a subset that is inconsistent but all of its proper subsets are consistent. For example, $\{p, q, \neg(p \vee q)\}$ is a minimal inconsistent set (under the assumption that $\{p, q, \neg p, \neg q, p \vee q, \neg(p \vee q)\}$ is the agenda).

The condition of value restriction is based on the minimal inconsistent sets of the agenda; it postulates that for every minimal inconsistent subset of the agenda, there exist two formulas in the set such that no agent accepts both.

Definition 23 (Dietrich and List, 2010)**.** *A profile is value-restricted if every minimal inconsistent set $Y \subset X$ has a two-element subset $Z \subseteq Y$ that is not a subset of any A_i.*

If we regard the union of attacks as the agenda and regard a cycle as a minimal inconsistent set, then Definition 22 is a restatement of Definition 23: for every cycle (minimal inconsistent subset) of the union of attacks (agenda), there exist two attacks such that no agent supports both.

Proposition 27 (Dietrich and List, 2010)**.** *For any profile* $(A_1, \cdots, A_n)$ *of consistent judgment sets, if* $(A_1, \cdots, A_n)$ *is value-restricted, then the majority outcome is consistent.*

Thus, Theorem 26 is a translation of Proposition 27, as both conditions are sufficient to ensure consistent majority outcomes or to preserve the basic semantic property of admissibility.

Finally, we compare the topological restrictions of t-self-defense and t-attack with value restriction. We present a definition of value restriction for binary aggregation given by Colley [24]. In the context of judgment aggregation, this definition reads as follows.

Definition 24 (Colley, 2019)**.** *A profile is value-restricted with respect to a constraint* Γ *if and only if for all prime implicates* π *of* Γ *containing two or more literals, there exist two distinct literals* ℓ_i *and* ℓ_j *of* π *such that no voter disagrees with both of them.*

A constraint is a formula. We can think of the constraints as being in *conjunctive normal form* (CNF), meaning that each is expressed as a conjunction of clauses. A clause is a disjunction of literals. Definition 24 reflects a particular kind of agreement among agents: given an integrity constraint Γ, for every prime implicate π of Γ, there exists a specific pair of literals in the prime implicate such that no individual disagrees with both of them.

If we interpret the structural property associated with an attack as a constraint and a counterattack to the attacker as a literal, then the definition of t-self-defense (Definition 20) can be considered a translation of Definition 24. At first glance, Definition 20 is restrictive, as it requires that for all pairs of counterattacks, at least one of them is accepted by at least one agent. This is because this definition has been used to preserve the admissibility of all sets of arguments rather than the admissibility of a specific set of arguments. The definition of t-attack (Definition 21) can be translated from Definition 24 in a similar way.

8 Conclusion

In this paper, we have studied the preservation of semantic properties during the aggregation of argumentation frameworks with topological restrictions. The topological restrictions we consider in this paper include restrictions based on topological properties as well as restrictions generalizing social-choice-theoretic domain conditions, and the semantic properties we consider include conflict-freeness, admissibility, being an extension under specific semantics, nonemptiness of the grounded extension and coherence. In contrast to the preservation results without constraints presented for several semantic properties by Chen and Endriss, which show that only dictatorships preserve them, we show that there exist aggregation rules with intuitive appeal that preserve these semantic properties when suitable topological restrictions are imposed. When the restriction under consideration is t-self-defense, t-attack, or t-acy, it is even possible to preserve several semantic properties with the majority rule. Table 1 summarises these preservation results, note that the admissibility, grounded, complete, preferred and stable semantics, the property of coherence, acyclicity, will be denoted as Ad, Gr, Co, Pr, Coh, Stb and Acy, respectively. The nomination rule and the majority will be denoted as Nomn

	Ad	Gr	Co	Pr	Stb	Coh	Acy
acyclicity	–	Nomn	Nomn	Nomn	–	–	–
symmetry	Gr&N	–	–	Nomn	–	Gr&N	–
coherence	–	–	–	Nomn	–	–	–
determined-ness	–	Nomn	–	–	–	–	–
t-self-defense	Maj	–	–	–	–	–	–
t-attack	–	–	–	–	Maj	–	–
t-acy	–	–	–	–	–	–	Maj

Table 1: Summary of Preservation Results

and Maj respectively, and the rules that are grounded and neutral will be denoted as Gr&N. It is worth mentioning that we use "–" under a property to denote that we did not present any result for it in this paper, despite that there may already exist preservation results for such property without restrictions in the literature.

The reader may have noticed that the topological properties studied in Section 5 have a common feature: for every AF satisfying such a topological property, two or more semantics prescribe the same sets of extensions, i.e., these semantics are in agreement. This is an important point indicating an opportunity to study the topological properties that give rise to semantics agreement. With such properties, we can naturally transfer aggregation results from one semantic property to another, which makes it easy to obtain results for other properties, and hopefully, we can obtain more positive results during aggregation. Understanding the relation between topological restrictions and semantic property preservation will help us to pinpoint how we need to relax our requirements to make preservation possible after all.

While all of the topological-restriction conditions discussed in this paper are useful for obtaining AFs that satisfy certain desirable semantic properties, it is worth discussing some directions for future work. First, additional topological-restriction conditions can be studied in the future, including ones that have already been defined in the literature as well as ones that have not been mentioned explicitly but are nevertheless promising topological restrictions. For example, we may consider it irrational for an individual agent to support an argumentation framework that contains one or more odd-length cycles, in which one argument may both indirectly attack and support another. In such a case, the acceptance status of the second argument is controversial with respect to the first, and we would like to avoid such controversy. Accordingly, we can require that the agents' argumentation frameworks satisfy the condition of excluding odd-length cycles.

The second possible direction of future research is to study the incentives for manipulating individuals in the aggregation process. Unlike acyclicity, symmetry, coherence, and determinedness, the topological restrictions of t-self-defense, t-attack and t-acy are not independent conditions, in the sense that the domain of the *AF*s than an individual can submit may depend on the *AF*s submitted by others. If we interpret an aggregation problem as a game, in which the individuals' possible inputs, i.e., their choices of the attack-relations are their strategies, then we can study the incentives for an agent to manipulate other individuals, i.e., by submitting an insincere *AF* to the aggregation rule. For example, let us consider the topological restriction of t-acy and the semantic property of acyclicity and consider a profile in which agent 1 submits $\{A \rightharpoonup B, B \rightharpoonup C\}$, agent 2 submits $\{B \rightharpoonup C, C \rightharpoonup A\}$, and agent 3 submits $\{A \rightharpoonup B, C \rightharpoonup A\}$. The majority rule does not preserve acyclicity in this case. However, if agent 3 wants to preserve acyclicity, she can submit an insincere *AF* that contains only $A \rightharpoonup B$; then, the new profile will be topologically restricted with respect to t-acy, and the majority rule will consequently preserve acyclicity.

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