Guaranteeing Admissibility of Abstract Argumentation Frameworks with Rationality and Feasibility Constraints

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Abstract

This paper considers the problem of under what circumstances an aggregation rule guarantees admissible sets of arguments that represents a good compromise between several extensions, i.e., sets of arguments of abstract argumentation frameworks each provided by a different individual. We start by showing that the preservation results of Dung's admissibility, graded admissibility, and strong admissibility during the aggregation of extensions are negative. To overcome such negative results, we define a model for extension aggregation that clearly separates the constraint supposed to be satisfied by individuals and the constraint that must be met by the collective decision. Using this model, we show that the majority rule guarantees admissible sets on profiles that satisfy a variant of Dung's admissibility, as well as profiles of extensions with some specific characteristics.

Keywords— argumentation theory, judgment aggregation, admissibility, value restriction

1 Introduction

Admissibility is an important semantic property of argumentation frameworks. It lies at the heart of all semantics discussed in [17] and is shared by many more recent proposals [3]. Under Dung's argumentation framework [17], a set of arguments satisfies admissibility if it defends all its members in the sense that for any argument A in the set, either A is un-attacked or if attacked by some argument B, then there is an argument in the set that attacks B, and it does not contain internal attacks.

When a group of agents is confronted with the same abstract argumentation framework, and each of them chooses an *extension* (a set of arguments), we may wish to aggregate such extensions into a collective one, which represents the consensus of the group. Similar questions have received attention in the recent decades (see, e.g., [29, 9, 1, 7, 11]). In this paper, we address the question of under what circumstances, an aggregation rule will guarantee admissible outcomes during the aggregation of extensions of abstract argumentation framework. In the existing literature, we note that Chen and Endriss [11] have shown that no aggregation rule preserves Dung's admissibility in general.

Graded semantics is a new theory of justification of arguments developed by Grossi and Modgil [22], in which the degree of acceptance of arguments can be weakened or strengthened. In graded semantics, the number of attackers and defenders are given a fine-grained assignment when deciding whether a specific argument is acceptable. While preserving Dung's admissibility is difficult, there is still no good news for the preservation of graded admissibility. Our results show that no quota rule guarantees admissible outcomes on profiles of graded admissible sets.

Strong admissibility is another notion of admissibility proposed by Baroni and Giacomin [2]. The motivation of strong admissibility is to characterize the unique properties of the grounded extension, a set of acceptable arguments about which there is no ambiguity. Our result demonstrates that the nomination rule, a rule granting some agents veto power, preserves this property.

In this paper, we focus on guaranteeing admissible outcomes when all agents' choices on acceptable arguments are admissible in the current argumentation framework.[1] We introduce a new model for extension aggregation. In nearly all existing literature on extension aggregation, there is only a single type of constraint (see, e.g., [29, 11]). Such a constraint is explicitly represented or left implicit. Following the model proposed by Endriss [19] for judgment aggregation [23, 18], we introduce a model for extension aggregation that allows the constraints assumed to be satisfied by the individual agents to be different from the constraints met by the collective decision. Consider the following example, which is adapted from [19].

Example 1. A university council with 5 members needs to decide on the funding for three projects: (φ_1): refurbishing the university stadium, (φ_2): organising an international conference, (φ_3): building a new student dormitory. The budget is limited, and it is not feasible to fund all three projects. However, the councilors are not required to keep this constraint in mind when casting their votes on the projects. Instead, they are assumed to support at least one of the projects, i.e., it would be irrational for a councilor not to recommend any of the projects for funding. Suppose their votes are as shown in Table 1.

	φ_1	φ_2	φ_3
Councillor 1	1	1	0
Councillor 2	0	0	1
Councillor 3	1	0	1
Councillor 4	1	1	0
Councillor 5	1	1	1

Table 1: Scenario used in Example 1

However, despite all five councillors being rational, i.e., all voters support at least one of the projects, at least three of them believe that the university stadium should be refurbished, the international conference should be organised, and the student dormitory should be built, a violation of the feasibility constraint as the university is not able to fund all three projects. Thus, every councillor's vote meets the rationality constraint. However, the outcome of the majority rule violates the feasibility constraint. △

We make use of the concept of solid admissibility [26] in this paper. A set of arguments is solidly

[1] There are at least five concepts of admissibility in this paper, including Dung's admissibility, graded admissibility, strong admissibility, weak admissibility and solid admissibility. Notably, when we refer to admissibility, we are referring to Dung's admissibility. In other words, unless stated otherwise, we are talking about Dung's admissibility.

admissible if it contains no arguments that attack themselves or other arguments in the set and contains all counterattacks against all attacks of arguments in the set. We say that an argument A is solid acceptable with respect to a set of arguments Δ if Δ includes all defenders of A, i.e., for any attacker B of argument A, Δ includes all attackers of B. By comparison, Dung's admissibility requires that Δ includes only one attacker of B. Using the model with rationality and feasibility constraints, we show that the majority rule guarantees admissible outcomes on profiles of solidly admissible sets. While the majority rule guarantees admissible outcomes on solidly admissible sets, a more specific approach is domain restriction, or restricting the domain of inputs in the sense of allowing only profiles with some specific characteristic. We show that if a profile of extensions with a specific characteristic satisfies the constraints that imply admissibility, then the majority rule guarantees admissible outcomes.

Related work The problem of aggregation of extensions of argumentation frameworks is extensively studied in the literature. Bodanza et al. [6] provide a survey on collective argumentation recently. Chen and Endriss [11] have studied the preservation of semantic properties by making use of quota rules, a family of aggregation rules that is conceptually and computationally simple. Rahwan and Tohmé [29] observe that "argument-wise plurality rule", a rule that is similar in spirit to the majority rule, does not preserve the completeness of labellings. Caminada and Pigozzi [9] design aggregation rules to obtain meaningful labellings. This line of work has been continued by Awad et al. [1], who present more impossibility results, and disscuss the possibility of obtaining favourable results by restricting the domain of the input.

Most of existing work on extension aggregation deals with a single type of constraint that defines what individual choices on arguments are permissible. For example, Chen and Endriss [11] assume that the input and the output need to satisfy the same requirements during aggregation of extensions, i.e., whether the semantic properties shared by all extensions reported by the individual agents will be preserved. A notable exception is the work by Caminada and Pigozzi [9], they focus on designing aggregation operators to obtain conflict-free, admissible, or complete labellings. Notably, for the output labellings it is not necessarily to impose the same restrictions as for the input labellings. Thus, using my terminology, the feasibility constraints can possibly be different from the rationality constraints. The idea of obtaining meaningful labellings is closely related to the idea of guaranteeing feasible outcomes. Endriss [19] provides a model for judgment aggregation that separates rationality constraints from feasibility constraints. Even though abstract argumentation is a possible domain of application for such a model of judgment aggregation, no result related to argumentation has been presented.

The idea of restricting domain of input has been discussed by Awad et al. [1]. With domain restriction, only profiles that satisfy certain restriction are allowed to be taken as the input. Notably, Awad et al. [1] have shown that the argument-wise plurality rule would be guaranteed to produce collectively rational outcomes on argumentation frameworks with graph-theoretical restrictions. This result may be considered conceptually—albeit not techincal—generalisation of our results regarding value restriction. Colley [13] defines value restriction for judgment aggregation, for which the original idea comes from domain restriction by Dietrich and List [16]. Notably, we will make use of results on a model of judgment aggregation by Colley [13] .

It is worth noting that there is much more work that applies the methodology of social choice theory in the context of abstract argumentation. However, such work is dedicated to deal with the problem of aggregating alternative argumentation frameworks (see, e.g., [14, 31, 15, 12]). In contrast, this work deals with the scenarios where every agent faces the same argumentation framework and chooses different extensions.

Paper overview The rest of this paper is organised as follows. In Section 2, we review some of Dung's basic concepts of the theory of abstract argumentation. Section 3 recalls the preservation results of Dung's admissibility by Chen and Endriss [11]. In Section 4, we show that preserving graded semantics yields similar impossibility results. In Section 5, we show preservation results for strong admissibility. In Section 6, we introduce a new model with rationality and feasibility constraints and illustrate a positive result with majority rule for solid admissibility. In Section 7, we introduce value restriction to guarantee admissibility with rationality and feasibility constraints. We conclude in Section 8 by outlining some future research directions.

2 Abstract Argumentation

2.1 Abstract Argumentation Framework

In this section, we recall some of the fundamentals of the model of abstract argumentation first introduced by [17]. An *argumentation framework* is a pair $AF = \langle Arg, \rightharpoonup \rangle$, where Arg is a finite set of arguments and $\rightharpoonup$ is a binary relation on Arg. We say that A *attacks* B if $A \rightharpoonup B$ holds for two arguments $A, B \in Arg$. For $\Delta \subseteq Arg$ and $B \in Arg$, we write $\Delta \rightharpoonup B$ (namely, Δ attacks B) in case $A \rightharpoonup B$ for at least one argument $A \in \Delta$. For $\Delta \subseteq Arg$ and $C \in Arg$, we say that Δ *defends* C in the case that Δ attacks all arguments $B \in Arg$ with $B \rightharpoonup C$. We write 2^{Arg} for the powerset of Arg.

Given an argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$, the question arises which subset Δ of the set of arguments Arg to accept. Any such set $\Delta \subseteq Arg$ is called an *extension* of AF. Different criteria have been proposed for choosing an extension. While Dung has defined several semantics, notably complete, grounded, preferred, and stable semantics [17], it is worth mentioning that conflict-freeness, being self-defending, and admissibility are fundamental properties supposed to be satisfied by extensions of semantics.

Definition 1. *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework and let $\Delta \subseteq Arg$ be a set of arguments. We adopt the following terminology:*

- *Δ is called conflict-free if there are no arguments $A, B \in \Delta$ such that $A \rightharpoonup B$.*
- *Δ is called self-defending if $\Delta \subseteq \{C \mid \Delta \text{ defends } C\}$.*
- *Δ is called admissible if it is both conflict-free and self-defending.*

Thus, a set of arguments is admissible if it is both conflict-free and self-defending. Furthermore, a set of arguments $\Delta \subseteq Arg$ is a complete extension if it is admissible and includes all arguments it defends, Δ is a preferred extension if it is a maximal complete extension, Δ is the grounded extension if it is the minimal complete extension, and finally, Δ is a stable extension if it is admissible and attacks every argument in $Arg \backslash \Delta$. All of them are admissibility-based in the sense that every extension of such semantics is admissible.

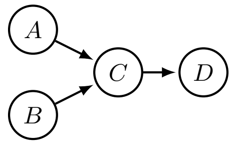

Figure 1: An argumentation framework with four arguments

2.2 Abstract Argumentation Semantics and Propositional Logic

Following the work by Besnard and Doutre [5] and Chen and Endriss [11], we represent the properties of extensions in a purely syntactic manner using a logical language. Fix an argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$, we can think of Arg as a set of propositional variables, and let $\mathcal{L}_{AF}$ be the corresponding propositional language. Now, extensions $\Delta \subseteq Arg$ correspond to models of formulas in $\mathcal{L}_{AF}$:

- $\Delta \models A$ for $A \in Arg$ if and only if $A \in \Delta$
- $\Delta \models \neg\varphi$ if and only if $\Delta \models \varphi$ is not the case
- $\Delta \models \varphi \wedge \psi$ if and only if both $\Delta \models \varphi$ and $\Delta \models \psi$

The semantics of disjunction is defined by $\Delta \models \varphi \vee \psi$ if and only if $\Delta \models \neg(\neg\varphi \wedge \neg\psi)$, and implication is defined by $\Delta \models \varphi \rightarrow \psi$ if and only if $\Delta \models (\neg\varphi \vee \psi)$. For example, $\Delta \models A \vee \neg B$ if and only if $A \in \Delta$ or $B \notin \Delta$. Given a formula φ, we use $\mathrm{Mod}(\varphi) = \{\Delta \subseteq Arg \mid \Delta \models \varphi\}$ to denote the set of all models of φ. Every formula φ identifies a property of extensions of AF, namely, $\sigma = \mathrm{Mod}(\varphi)$. When using a formula φ to describe such a property of extensions, we usually refer to φ as an *integrity constraint*. The following simple result characterises the properties of being conflict-free and self-defending in terms of integrity constraints expressed in $\mathcal{L}_{AF}$.

Proposition 1. *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework and let $\Delta \subseteq Arg$ be an extension. Then, Δ is conflict-free if and only if:*

$$\Delta \models \mathrm{IC}_{CF} \quad \textit{where} \quad \mathrm{IC}_{CF} \;=\; \bigwedge_{\substack{A,B \in Arg \\ A \rightharpoonup B}} (\neg A \vee \neg B)$$

That is, Proposition 1 states that $\mathrm{Mod}(\mathrm{IC}_{CF}) = \{\Delta \subseteq Arg \mid \Delta$ *is conflict-free*$\}$.

Proposition 2. *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework and let $\Delta \subseteq Arg$ be an extension. Then, Δ is self-defending if and only if:*

$$\Delta \models \mathrm{IC}_{SD} \quad \textit{where} \quad \mathrm{IC}_{SD} \;=\; \bigwedge_{C \in Arg} [C \rightarrow \bigwedge_{\substack{B \in Arg \\ B \rightharpoonup C}} \bigvee_{\substack{A \in Arg \\ A \rightharpoonup B}} A]$$

We can now use the integrity constraints defined above to construct integrity constraints for the property of admissibility:

Proposition 3. *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework and let $\Delta \subseteq Arg$ be an extension. Then, Δ is admissible if and only if $\Delta \models \mathrm{IC}_{AD}$, where $\mathrm{IC}_{AD} = \mathrm{IC}_{CF} \wedge \mathrm{IC}_{SD}$.*

Example 2. Consider the argumentation framework $AF = \langle \{A, B, C, D\}, \{A \rightharpoonup C, B \rightharpoonup C, C \rightharpoonup D\} \rangle$, as illustrated in Figure 1. Then $\mathrm{IC}_{SD} = (\neg D \vee A \vee B) \wedge (\neg C)$, $\mathrm{IC}_{AD} = (\neg D \vee A \vee B) \wedge (\neg C) \wedge (\neg A \vee \neg C) \wedge (\neg B \vee \neg C) \wedge (\neg C \vee \neg D)$. $\triangle$

3 Preservation of Dung's Admissibility

3.1 A Model with a Single Constraint

In this section, we recall the model for extension aggregation defined by Chen and Endriss [11]. Such a model allows a single type of constraint. Fix an argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$, and let $U = \{1, \ldots, u\}$ be a finite set of *agents*. Suppose each agent $i \in U$ supplies us with an extension $\Delta_i \subseteq Arg$, reflecting her individual views of what constitutes an acceptable set of arguments in the context of AF. Thus, we are supplied with a *profile* $\mathbf{\Delta} = (\Delta_1, \ldots, \Delta_u)$, i.e., a vector of extensions, one for each agent. An *aggregation rule* is a function $F : (2^{Arg})^u \to 2^{Arg}$ mapping any given profile of extensions to a single extension. Note that we use voter and agent interchangeably.

Definition 2. *A quota rule F_q with quota q is the aggregation rule mapping any given profile of extensions to the extension including exactly those arguments accepted by at least q agents.*

The quota rules have low computational complexity in the sense that the outcomes are straightforward to compute [20]. The nomination rule is the quota rule with quota $q = 1$. The majority rule is another example of a quota rule for which the quota $q = \lceil \frac{u+1}{2} \rceil$.

Definition 3 (Preservation)**.** *Let AF be an argumentation framework. Given a semantic property $\delta \subseteq 2^{Arg}$ and an aggregation rule $F : (2^{Arg})^u \to 2^{Arg}$ for u agents, we say that F preserves δ if for all profiles $\mathbf{\Delta} = (\Delta_1, \ldots, \Delta_u) \in \delta^u$, we have $F(\mathbf{\Delta}) \in \delta$.*

Thus, given a property δ, if all extensions reported by agents satisfy δ, the concept of preservation says that we would like the outcome of the aggregation rule to satisfy δ as well.

We can see that only a single property appears in the above definition. We will refer to a property as a constraint; thus, the definition represents the concept of preservation of properties with respect to a single constraint. In this paper, we focus on the property of admissibility. We first consider three concepts of admissibility, namely, Dung's admissibility, graded admissibility, and strong admissibility, and study the preservation results for them.

3.2 Preservation of Dung's Admissibility

Chen and Endriss [11] have considered the problem of the aggregation of alternative extensions by making use of quota rules. They exploit known encodings of argumentation semantics in propositional logic and study the preservation of semantic properties of extensions, including conflict-freeness, being self-defending, admissibility, and other classical semantics. Here, we recall the preservation results for admissibility, as well as the results for conflict-freeness and being self-defending, two constituent properties of admissibility.

Theorem 4 (Chen and Endriss, 2018)**.** *Let AF be any argumentation framework with at least one attack between two arguments that do not attack themselves. Then, a quota rule F_q for u agents preserves conflict-freeness of AF if and only if $q > \frac{u}{2}$.*

Thus, the (strict) majority rule preserves conflict-freeness, as does a quota rule with an even higher quota. For the preservation of being self-defending, we first present a technical lemma.

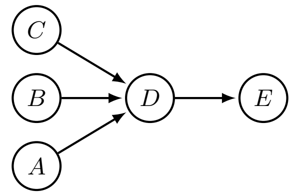

Figure 2: An argumentation framework with five arguments

Lemma 5 (Chen and Endriss, 2018)**.** *A quota rule F_q for u agents preserves the property of being self-defending for an argumentation framework AF if* $q \cdot (\text{MaxDef}(AF) - 1) < \text{MaxDef}(AF)$*.*

Note that $\text{MaxDef}(AF)$ is the maximum number of attackers of an argument that itself is the source of an attack. This lemma suggests that the preservation is possible for low quotas.

Proposition 6 (Chen and Endriss, 2018)**.** *The nomination rule preserves the property of being self-defending.*

Proposition 7 (Chen and Endriss, 2018)**.** *Every quota rule F_q for u agents with a quota $q > \frac{u}{2}$ preserves admissibility for all argumentation frameworks AF with* $\text{MaxDef}(AF) \leqslant 1$*.*

If we remove the assumptions regarding the structure of the argumentation framework (such as $\text{MaxDef}(AF) \leqslant 1$), the following impossibility result shows that the conflict regarding the quotas between being self-defending and conflict-freeness cannot be resolved in general.

Theorem 8 (Chen and Endriss, 2018)**.** *No quota rule preserves admissibility for all argumentation frameworks.*

Thus, no quota rule guarantees admissibility in general. The majority rule, as an example of a quota rule, fails to preserve admissibility as well, as the following example shows:

Example 3. Let $AF = \langle Arg, \rightharpoonup \rangle$ with $Arg = \{A, B, C, D, E\}, (\rightharpoonup) = \{A \rightharpoonup D, B \rightharpoonup D, C \rightharpoonup D, D \rightharpoonup E\}$. This AF is illustrated in Figure 2. Suppose three agents evaluate the argumentation framework, they report the extensions $\{A, E\}$, $\{B, E\}$, and $\{C, E\}$, respectively, all of which are admissible. However, applying the majority rule yields $\{E\}$, which is not admissible. Thus, the majority rule fails to preserve Dung's admissibility. △

4 Preservation of Graded Admissibility

4.1 Graded Admissibility

In this part, we present graded admissibility introduced by Grossi and Modgil [22]. Graded admissibility is fundamental to graded semantics, which is a generalization of Dung's semantics. Extensions of graded semantics are weakened or strengthened depending on the levels of being self-defending and conflict-freeness they satisfy.

Given an argument $A \in Arg$ and a set of arguments $\Delta \subseteq Arg$, recall that under Dung's standard semantics, we say that A is defended by Δ if whenever A is attacked by some argument $B \in Arg$, there is at least one argument in Δ that attacks B. Grossi and Modgil generalize this notion to obtain the notion of *graded defense* [22].

Definition 4 (Dung, 1995)**.** *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework, the defense function is defined as follows. For any $\Delta \subseteq Arg$:*

$$\mathrm{d}(\Delta) = \{X \in Arg \mid \forall Y \in Arg: \text{ IF } Y \rightharpoonup X \text{ THEN } \Delta \rightharpoonup Y\} \tag{1}$$

Definition 5 (Grossi and Modgil, 2019)**.** *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework, and let m and n be two positive integers $(m, n > 0)$. The graded defense function for Δ is defined as follows. For any $\Delta \subseteq Arg$:*

$$\mathrm{d}_n^m(\Delta) = \left\{X \in Arg \ s.t. \ |\{Y \in \overline{X} \ s.t. \ |\overline{Y} \cap \Delta| < n\}| < m\right\} \tag{2}$$

where $\bar{X}$ denotes $\{Y \in Arg \mid Y \rightharpoonup X\}$.

Thus, $\mathrm{d}_n^m(\Delta)$ denotes the set of arguments that have at most $m-1$ attackers that are not counter-attacked by at least n arguments in Δ.

Example 4. Let us consider the argumentation framework $AF = \langle \{A, B, C, D\}, \{A \rightharpoonup C, B \rightharpoonup C, C \rightharpoonup D\}\rangle$, as depicted in Figure 1. Let $\Delta = \{A, D\}$, it is easy to verify that $D \in \mathrm{d}_1^1(\Delta)$ but $D \notin \mathrm{d}_2^1(\Delta)$. $\triangle$

Definition 6 (Grossi and Modgil, 2019)**.** *A set of arguments Δ is said to be acceptable at grade mn (or mn-acceptable) whenever all of its arguments are such that at most $m-1$ of their attackers are not counter-attacked by at least n arguments in Δ.*

Definition 7 (Grossi and Modgil, 2019)**.** *A set of arguments Δ is said to be mn-self-defending whenever all of its arguments are such that at most $m-1$ of their attackers are not counter-attacked by at least n arguments in Δ.*

Definition 8 (Grossi and Modgil, 2019)**.** *A set of arguments Δ is said to be at grade mn-admissible whenever Δ is mn-acceptable and conflict-free.*

Thus, mn-admissibility is a collection of properties, each specified by two numbers m and n. In fact, when $m = n = 1$, we recover the standard definition of being self-defending and admissibility. Notably, Grossi and Modgil define graded admissibility as mn-acceptability plus ℓ-conflict-freeness (a set of arguments Δ is said to be ℓ-conflict-free whenever no argument $A \in \Delta$ such that A is attacked by at least ℓ arguments in Δ [22]). However, for the sake of simplicity, we restrict graded admissibility to mn-acceptability plus Dung's notion of conflict-freeness, a desirable subset of graded admissibility.

4.2 Preservation Result For Graded Admissibility

In this section, we present the results for the preservation of graded admissibility. We start by encoding the property of being graded self-defending in propositional logic and show a preservation result for it. We then present a result for the property of graded admissibility. The following simple result characterises the property of being graded self-defending in terms of integrity constraints expressed in $\mathcal{L}_{AF}$.

Proposition 9. *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework and let $\Delta \subseteq Arg$ be an extension. Then, Δ is mn-self-defending if and only if:*

$$\Delta \models \mathrm{IC}_{mnSD} \quad \textit{where} \quad \mathrm{IC}_{mnSD} = \bigwedge_{C \in Arg} [C \rightarrow \bigvee_{\{B_1,\ldots,B_{(|\bar{C}|-m+1)}\} \in \binom{|\bar{C}|}{|\bar{C}|-m+1}} (\bigwedge_{i=1}^{|\bar{C}|-m+1} (\bigvee_{\{A_1,\ldots,A_n\} \in \binom{|B_i|}{n}} (\bigwedge_{j=1}^{n} A_j)))] \tag{3}$$

Furthermore, Δ is mn-admissible if and only if $\Delta \models \mathrm{IC}_{mnAD}$, where $\mathrm{IC}_{mnAD} = \mathrm{IC}_{CF} \wedge \mathrm{IC}_{mnSD}$. To obtain the preservation results for being graded self-defending, we need a result regarding binary aggregation with integrity constraints [21], a variant of judgment aggregation.

Lemma 10 (Grandi and Endriss, 2013)**.** *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework, and let φ be a clause in $\mathcal{L}_{AF}$ with k_1 positive literals and k_2 negative literals. Then, a quota rule F_q for u agents preserves the property* $\mathrm{Mod}(\varphi)$ *if and only if:*

$$q \cdot (k_2 - k_1) > u \cdot (k_2 - 1) - k_1 \tag{4}$$

Note that a clause is a disjunction of literals and that all integrity constraints can be translated into conjunctions of clauses. The following result shows that if we know the preservation result for some clauses, then we know the results for the conjunction of such clauses.

Lemma 11 (Grandi and Endriss, 2013)**.** *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework, let φ_1 and φ_2 be integrity constraints in $\mathcal{L}_{AF}$, and let F be an aggregation rule that preserves both* $\mathrm{Mod}(\varphi_1)$ *and* $\mathrm{Mod}(\varphi_2)$*. Then, F also preserves* $\mathrm{Mod}(\varphi_1 \wedge \varphi_2)$*.*

Thus, given a quota rule F_q and some clauses $\varphi_1, \ldots \varphi_\ell$, if F_q satisfies all clauses φ_i, then it preserves $\mathrm{Mod}(\varphi_1 \wedge \cdots \wedge \varphi_\ell)$.

Example 5. Given an integrity constraint $\varphi = (\neg A \vee \neg B) \wedge C$. By Lemma 10, a quota rule preserves $\neg A \vee \neg B$ only if $q \cdot (2-0) > u \cdot (2-1) - 0$, i.e., only if $q > \frac{u}{2}$. A quota rule preserves C only if $q \cdot (0-1) > u \cdot (0-1) - 1$, which is always the case, thus, C is preserved by every quota rule. Therefore, according to Lemma 11, a quota rule preserves φ only if $q > \frac{u}{2}$. △

Recall that the nomination rule is the quota rule for which the quota is 1.

Proposition 12. *The nomination rule preserves the property of being an mn-self-defending set for all $m, n \in N$.*

Proof. Recall that IC_{mnSD} is a conjunction of formulas of the form

$$C \rightarrow \bigvee_{\{B_1,\ldots,B_{(|\bar{C}|-m+1)}\} \in \binom{|\bar{C}|}{|\bar{C}|-m+1}} (\bigwedge_{i=1}^{|\bar{C}|-m+1} (\bigvee_{\{A_1,\ldots,A_n\} \in \binom{|B_i|}{n}} (\bigwedge_{j=1}^{n} A_j)))$$

which can be rewritten as

$$C \rightarrow \bigwedge_{B_1,\ldots,B_m \in \binom{|\bar{C}|}{m}} [\bigvee_{i=1}^{c_1} (A_{\pi_i(1)} \wedge \cdots \wedge A_{\pi_i(n)})_1] \vee \cdots \vee [\bigvee_{i=1}^{c_m} (A_{\pi_i(1)} \wedge \cdots \wedge A_{\pi_i(n)})_m], \tag{5}$$

where $c_i = \binom{|\bar{B}_i|}{n}$ for $i = 1, \cdots, m$, respectively. We take one such clause

$$C \rightarrow [\bigvee_{i=1}^{c_1} (A_{\pi_i(1)} \wedge \cdots \wedge A_{\pi_i(n)})] \vee \cdots \vee [\bigvee_{i=1}^{c_m} (A_{\pi_i(1)} \wedge \cdots \wedge A_{\pi_i(n)})], \tag{6}$$

which can be rewritten as

$$C \rightarrow \bigwedge_{j=1}^{n} [\bigvee_{i=1}^{m} (A_{\pi_i(j)} \vee \cdots \vee A_{\pi_{(|\bar{B}_j|-n)}(j)})]. \tag{7}$$

We take one such clause

$$C \rightarrow [\bigvee_{i=1}^{m} (A_{\pi_i(j)} \vee \cdots \vee A_{\pi_{(|\bar{B}_j|-n)}(j)})]. \tag{8}$$

The number of positive literals is $(|\bar{B}_j| - n) \cdot m$, and the number of negative literals is 1. Thus, according to Lemma 10, a uniform quota rule with quota q preserves it if and only if

$$q < \frac{(|\bar{B}_j| - n) \cdot m}{(|\bar{B}_j| - n) \cdot m - 1}. \tag{9}$$

As IC_{mnSD} is a conjunction of such clauses, we need to satisfy this inequality for all relevant m, n and B_j values. This requirement is most demanding for the largest value of n, and the smallest of m and B_j. However, we note that if $q = 1$, then $q < \frac{(|\bar{B}_j|-n)\cdot m}{(|\bar{B}_j|-n)\cdot m-1}$ is always the case. Thus, we have the proposition. □

Note that an *mn*-admissible set is also *mn*-self-defending. As the nomination rule is well-behaved during the aggregation of self-defending sets, we may wonder whether there are reasonable aggregation rules that preserve *mn*-admissibility for all argumentation frameworks.

Theorem 13. *No quota rule preserves mn-admissibility for all argumentation frameworks for all* $m, n \in N$.

Proof. Recall that the standard definition of admissibility is a special case of *mn*-admissibility for which $m = n = 1$. By Theorem 8, we obtain that no quota rule preserves 11-admissibility. Thus, we have the theorem. □

Therefore, we obtain a negative result for *mn*-admissibility. Notably, graded admissibility is a collection of properties of extensions. For example, Dung's admissibility is an example of graded admissibility when $m = n = 1$. One may wonder whether there are some cases in which *mn*-admissibility can be preserved by some specific quota rules for all argumentation frameworks. The answer is no: we return to Proposition 12, in (9). If $|\bar{B}_j|$ is sufficiently large, then only the nomination rule preserves (8); therefore, only the nomination preserves the property of being *mn*-self-defending regardless of the values of m and n. While the nomination rule will not preserve conflict-freeness, given two numbers m and n, no quota rule preserves *mn*-admissibility.

5 Preservation of Strong Admissibility

The concept of strong admissibility was first introduced by Baroni and Giacomin [2] to capture the intrinsic properties of grounded semantics. In the spirit of strong admissibility, the notion of defense is rooted in the empty set, and every argument in the set is defended by a strongly admissible set that does not include an argument it defends.

Definition 9. *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework, we say that an argument $A \in Arg$ is strongly defended by Δ if and only if for each attacker $B \in Arg$ of A, B is attacked by some argument $C \in \Delta \backslash \{A\}$ and, in turn, C is strongly defended by $Arg \backslash \{A\}$.*

Definition 10. *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework, we say that a set of arguments $\Delta \subseteq Arg$ is a strongly admissible set if for every $A \in \Delta$, A is strongly defended by $\Delta \backslash A$, which in turn, is again strongly admissible.*

Theorem 14 (Caminada and Dunne, 2019)**.** *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework and let $\Delta \subset Arg$ be a strongly admissible set, then, Δ is conflict-free and admissible.*

Thus, we can think of admissible sets that are not strongly admissible. To illustrate this result, let us consider an argumentation framework that contains only two arguments A and B, which attack each other. Thus, both $\{A\}$ and $\{B\}$ are admissible, but neither of them is strongly admissible. Let us illustrate the notion of strong admissibility with an example adapted from Caminada and Dunne [8]

Example 6. Consider an argumentation framework $AF = \langle \{A, B, C, D, E, F, G, H\}, \{A \rightharpoonup B, B \rightharpoonup C, C \rightharpoonup E, D \rightharpoonup E, E \rightharpoonup F, F \rightharpoonup E, G \rightharpoonup H, H \rightharpoonup G, H \rightharpoonup B\}\rangle$, as illustrated in Figure 3. The strongly admissible sets of AF are $\emptyset, \{A\}, \{A, C\}, \{A, C, F\}, \{D\}, \{A, D\}, \{A, C, D\}, \{D, F\}, \{A, D, F\}$, and $\{A, C, D, F\}$. We consider the set $\{A, C, F\}$, which is strongly admissible as A is defended by $\emptyset$, C is defended by $\{A\}$, and F is defended by $\{A, C\}$, i.e., every member of $\{A, C, F\}$ is defended by a strongly admissible subset of $\{A, C, F\}$ that does not contain the argument it defends. However, as a subset of $\{A, C, F\}$, even though $\{A, F\}$ defends C, it is not strongly admissible. Furthermore, $\{C, H\}$ is admissible, but not strongly admissible, since no subset of $\{C, H\} \backslash \{H\}$ defends H. $\triangle$

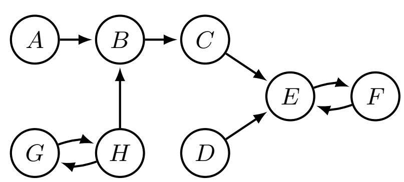

Figure 3: Scenario used in Example 6

Theorem 15 (Caminada and Dunne, 2019)**.** *Given two sets of argument $\Delta_1, \Delta_2 \subseteq Arg$, if Δ_1 and Δ_2 are strongly admissible sets, then $\Delta_1 \cup \Delta_2$ is also a strongly admissible set.*

We now turn to consider the preservation of strong admissibility. Following the model defined in Section 3.1, we assume that each agent reports a strongly admissible set, and we aggregate such sets of arguments into a collective set. We are interested in what aggregation rules will preserve the property of strong admissibility. The first aggregation rule that comes to our mind is the majority rule.

Example 7. Consider $AF = \langle \{A, B, C, D, E\}, \{A \rightharpoonup D, B \rightharpoonup D, C \rightharpoonup D, D \rightharpoonup E\}\rangle$, as illustrated in Figure 2. All of $\{A, E\}$,$\{B, E\}$,$\{C, E\}$ are strongly admissible. Nevertheless, the outcome of the majority rule, namely, $\{E\}$, is not strongly admissible. Thus, the majority rule fails to preserve strong admissibility. △

Thus, the majority rule fails to preserve strong admissibility. Recall that in Theorem 15, we know that the union of any pair of strongly admissible sets satisfies strong admissibility. Thus, another natural candidate for the aggregation procedure is the nomination rule, a quota rule F_q with quota $q = 1$.

Theorem 16. *Let $AF = \langle Arg, \rightharpoonup\rangle$ be any argumentation framework, then the nomination rule preserves strong admissibility for AF.*

Proof. Let F be the nomination rule. Take a profile of strongly admissible sets $\boldsymbol{\Delta} = (\Delta_1, \ldots, \Delta_u)$ for u agents. Recall that the nomination rule is the quota rule F_q for which the quota $q = 1$. Thus, any argument that is supported by at least one agent will be accepted by the nomination rule. Now, it remains to show that $\Delta_1 \cup, \ldots, \cup \Delta_u$ is strongly admissible, i.e., the union of all sets of arguments proposed by individuals is a strongly admissible set.

We construct a set of arguments Δ as follows. Initially, set $\Delta = \emptyset$. Then, for every j from 1 to u in turn, let Δ be the union of Δ and Δ_j:

$$\Delta = \Delta \cup \Delta_j$$

Note that $\emptyset$ is trivially strongly admissible; therefore, both Δ and Δ_j are strongly admissible sets. According to Theorem 15, $\Delta \cup \Delta_j$ is also a strongly admissible set. We therefore arrive at a set of arguments Δ, which is a is strongly admissible set. This completes the proof. □

Thus, there is a quota rule that preserves strong admissibility, which is somewhat surprising as strong admissibility is a stronger property than Dung's admissibility.

6 Guaranteeing Admissibility on Solidly Admissible Sets

6.1 Solid Admissibility

Graded semantics provides a theory of degree of justification of arguments. Under graded semantics, the assignment of status to arguments is defined by the numbers of attackers and defenders. While graded semantics appeals to the numbers of attackers and defenders to define acceptability of arguments, in some scenarios, two arguments with different numbers of attackers and defenders may have similar features. Consider the following example.

Example 8. Let us consider two sets of arguments $\Delta_1 = \{C_1, A_1, A_2, A_3\}$ and $\Delta_2 = \{C_2, A_3, A_4\}$ in AF, as illustrated in Figure 4. The numbers of defenders of C_1 and C_2 are different: C_1 has three defenders and C_2 has two defenders. However, both of them are solidly defended in the sense that for every argument C in the set, if A defends C, then A is included in the set. Clearly, C_1 has three

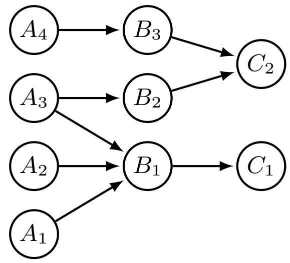

Figure 4: Scenario used in Example 8

defenders, namely, A_1, A_2, A_3, and they are included in Δ_1, while C_2 has two defenders, namely, A_3 and A_4, which are included in Δ_2. The similarity between C_1 and C_2 is not captured by Modgil and Grossi's graded acceptability. $\triangle$

Take an argument $A \in Arg$ and a set of arguments $\Delta \subseteq Arg$, under Dung's admissibility, we say that Δ defends A if for every attack $B \in Arg$ of argument A, Δ includes at least one attacker of B. We say that Δ is admissible if Δ defends all its members and being conflict-free. We now present the concept of *solid admissibility* introduced by Liu and Chen [26]. We say that Δ *solidly defends* A if Δ defends A and for every attacker B of arguments A, Δ accepts *all* attackers of B, i.e., Δ includes all defenders of A, we say that Δ satisfies solid admissibility if Δ solidly defends all its members and Δ is conflict-free.

Definition 11. *Take an argument $A \in Arg$ and a set of arguments $\Delta \subseteq Arg$. We say that Δ* solidly defends *A if (i) for every attacker B of A, there is an argument $C \in \Delta$ such that $C \rightharpoonup B$, and (ii) for every attacker B of A, Δ includes* all *attackers of B.*

Definition 12. *Take an argument $A \in Arg$ and a set of arguments $\Delta \subseteq Arg$. We say that Δ is* solidly admissible *if Δ solidly defends all its members, and Δ is conflict-free.*

Recall that we use φ to refer to a property of extensions or an integrity constraint. The following simple result characterises the property of being solid defending in terms of the integrity constraint expressed in $\mathcal{L}_{AF}$.

Proposition 17. *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework and let $\Delta \subseteq Arg$ be an extension. Then, Δ is solid defending if and only if:*

$$\Delta \models \mathrm{IC}_{SS} \quad \textit{where} \quad \mathrm{IC}_{SS} \;=\; \bigwedge_{C \in Arg} \bigwedge_{\substack{B \in Arg \\ B \rightharpoonup C}} [C \;\rightarrow ((\bigvee_{\substack{A \in Arg \\ A \rightharpoonup B}} A) \wedge (\bigwedge_{\substack{A \in Arg \\ A \rightharpoonup B}} A))]$$

We can now use the integrity constraint defined above to construct the integrity constraint for the property of solid admissibility:

- Δ is solidly admissible if and only if $\Delta \models \mathrm{IC}_{SA}$, where $\mathrm{IC}_{SA} = \mathrm{IC}_{CF} \wedge \mathrm{IC}_{SS}$.

Example 9. Consider the argumentation framework $AF = \langle \{A, B, C, D\}, \{A \rightharpoonup C, B \rightharpoonup C, C \rightharpoonup D\} \rangle$, as illustrated in Figure 1. Then, $\mathrm{IC}_{SD} = (\neg D \vee A \vee B) \wedge (\neg C)$, $\mathrm{IC}_{SS} = (\neg D \vee A) \wedge (\neg D \vee B) \wedge (\neg C)$. In this example, $\{A, D\}$ and $\{B, D\}$ are admissible, but they are not solidly admissible. $\{A, B, D\}, \{A\}, \{B\}, \{A, B\}, \emptyset$ are all admissible and solidly admissible sets. $\triangle$

6.2 A Model with Rationality and Feasibility Constraints

In nearly all existing literature on judgment aggregation [21, 25] and some work on extension aggregation [11], only a single type of constraint, namely, the *integrity constraint*, is considered. Integrity constraints decide what is permissible for both the input and the output. As we have shown in Section 3, Section 4, and Section 5, Dung's admissibility, graded admissibility, and strong admissibility fail to be preserved under a model that allows for integrity constraints only. In this section, we propose a new model that allows for a distinction between rationality and feasibility constraints.

We reuse terminology introduced in Section 3: let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework, and let $U = \{1, \cdots, u\}$ be a finite set of agents. Suppose that every agent provides an extension Δ_i, which gives rise to a profile of extensions $\boldsymbol{\Delta} = \{\Delta_1, \ldots, \Delta_u\}$. A profile is Γ-rational if $\Delta_i \models \Gamma$ for all $i \in U$. Thus, we use Γ to define the permissible profiles of extensions, which is called a *rationality constraint*. An outcome is Γ'-feasible if the outcome satisfies such a constraint. We call Γ' a *feasibility constraint*, which defines the acceptable outcomes.

Definition 13. *An aggregation rule $F : (2^{Arg})^u \to 2^{Arg}$ is said to guarantee Γ'-feasibility on Γ-rational profiles of extensions if for every profile $\boldsymbol{\Delta} \in \mathrm{Mod}(\Gamma)^u$, it is the case that $F(\boldsymbol{\Delta}) \in \mathrm{Mod}(\Gamma')$.*

This definition is a restatement of Definition 1 by Endriss [19] using our present terminology. Thus, we say that F guarantees Γ'-feasible outcomes on Γ-rational profiles if for any profile $\boldsymbol{\Delta}$ for which $\Delta_i \models \Gamma$ for all $i \in U$ is the case, we have $\Delta \models \Gamma'$.

6.3 Guaranteeing Admissibility on Solidly Admissible Sets

In this section, we present our results on guaranteeing admissibility on solidly admissible sets with the model with rationality and feasibility constraints. Before proceeding, we generalize the notion of *prime implicate* to our context. A clause $\pi \in \mathcal{L}_{AF}$ is a prime implicate of a formula $\Gamma \in \mathcal{L}_{AF}$ if (i) $\Gamma \models \pi$ and (ii) for every clause $\pi' \in \mathcal{L}_{AF}$ with $\Gamma \models \pi'$ if $\pi' \models \pi$, then $\pi = \pi'$ [27]. In other words, the prime implicates are the logically strongest clauses entailed by Γ. We return to Example 9, the integrity constraint for solid admissibility of AF is $\mathrm{IC}_{SS} = (\neg D \vee A) \wedge (\neg D \vee B) \wedge (\neg C)$. Then, the set of prime implicates of IC_{SS} includes $(\neg D \vee A)$, $(\neg D \vee B)$, and $(\neg C)$. In the literature of abstract argumentation, Moguillansky and Simari [28] study upon prime implicates as an ingredient for constructing a family of abstract argumentation frameworks, namely generalized argumentation frameworks, which are designed for bridging the gap between argumentation frameworks and logic-based argumentation systems.

Recall that a clause is a disjunction of literals. A clause is simple if it has at most two literals. A clause is nonsimple if it cannot be simplified to a clause with less than three literals. A formula is simple if it is logically equivalent to a conjunction of clauses with at most two literals (it is also called a Krom formula). We first present three results concerning prime implicates.

Fact 18. *A formula Γ is simple if and only if all its prime implicates are simple.*

For example, the integrity constraint for solid admissibility IC_{SS} is simple. To see this, recall from Proposition 17 that IC_{SS} is a conjunction of formulas of the form $C \rightarrow (\bigvee_{\substack{A \in Arg \\ A \rightharpoonup B}} \bigwedge_{\substack{A \in Arg \\ A \rightharpoonup B}} A)$: as an attacker of C, if B has at least one attacker, then this formula can be simplified to $C \rightarrow (\bigwedge_{\substack{A \in Arg \\ A \rightharpoonup B}} A)$, which we can rewrite as $(\neg C \vee A_1) \wedge \cdots \wedge (\neg C \vee A_l)$, in which l is the number of attackers of B; if

B does not have any attacker, then this formula can be simplified to $C \to \bot$. Thus, in all possible scenarios, this formula is a conjunction of clauses with at most two literals, i.e., it is simple. Then, according to Fact 18, all prime implicates of IC_{SS} are simple.

Lemma 19 (Marquis, 2000)**.** *If $\Gamma \models \Gamma'$ is the case, then for every prime implicate π' of Γ', there exists a prime implicate π of Γ such that $\pi \models \pi'$.*

Definition 14 (Endriss, 2018)**.** *A pair of formulas (Γ, Γ') is simple if for every nonsimple prime implicate π' of Γ', there exists a simple prime implicate π of Γ such that $\pi \models \pi'$.*

Using the results above, we are now ready to present some results concerning the relation between self-defending and solid defending and the relation between Dung's admissibility and solid admissibility.

Lemma 20. $\mathrm{IC}_{SS} \models \mathrm{IC}_{SD}$.

Proof. Recall that IC_{SD} is a conjunction of a collection of formulas of the form $C \to \bigwedge_{\substack{B \in Arg \\ B \rightharpoonup C}} \bigvee_{\substack{A \in Arg \\ A \rightharpoonup B}} A$. We rewrite it as $\bigwedge_{\substack{B \in Arg \\ B \rightharpoonup C}} (\neg C \vee \bigvee_{\substack{A \in Arg \\ A \rightharpoonup B}} A)$. This formula is a conjunction of a collection of clauses of the form $(\neg C \vee \bigvee_{\substack{A \in Arg \\ A \rightharpoonup B}} A)$. We denote it by φ'. Clearly, φ' is a clause of IC_{SD}. We are now going to show that there is at least one clause φ of IC_{SS} such that $\varphi \models \varphi'$. We recall that IC_{SS} is a conjunction of a collection of formulas of the form $\bigwedge_{\substack{B \in Arg \\ B \rightharpoonup C}} [C \to (\bigvee_{\substack{A \in Arg \\ A \rightharpoonup B}} \bigwedge_{\substack{A \in Arg \\ A \rightharpoonup B}} A)]$. Let us consider one formula $[C \to (\bigvee_{\substack{A \in Arg \\ A \rightharpoonup B}} \bigwedge_{\substack{A \in Arg \\ A \rightharpoonup B}} A)]$ indexed by $B \in Arg$ with $B \rightharpoonup C$. We denote it by φ.

If no argument attacks B, then both φ and φ' can be simplified to $\neg C$. Thus, we have found a clause of IC_{SS} such that $\varphi \models \varphi'$. We now consider the scenario that B has at least one attacker. In this scenario, φ can be rewritten as $(\neg C \vee A_1) \wedge \ldots \wedge (\neg C \vee A_n)$, in which $A_1, \ldots A_n$ defend C by attacking B as well. We denote this scenario by $\varphi_1 \wedge \ldots \wedge \varphi_n$. Since $A_1, \ldots A_n$ defend C by attacking B, we know that $(\neg C \vee A_i) \models (\neg C \vee A_1 \vee \ldots \vee A_n)$ for $i \in \{1, \ldots, n\}$. Thus, $\varphi_i \models \varphi'$ for $i \in \{1, \ldots, n\}$.

Using the same construction, we can show that for every clause φ' of IC_{SD}, there is at least one clause φ of IC_{SS} such that $\varphi \models \varphi'$. Thus, $\mathrm{IC}_{SS} \models \mathrm{IC}_{SD}$. □

Proposition 21. *$(\mathrm{IC}_{SS}, \mathrm{IC}_{SD})$ is simple.*

Proof. From Lemma 20, we know that $\mathrm{IC}_{SS} \models \mathrm{IC}_{SD}$. From Lemma 19, we know that for every prime implicate π' of IC_{SD}, there exists a prime implicate π of IC_{SS} such that $\pi \models \pi'$. Clearly, IC_{SS} is a conjunction of clauses with at most two literals. Thus, it is simple. By Fact 18, we have that every prime implicate of IC_{SS} is simple.

Putting together the above facts, we can conclude that for every (simple and nonsimple) prime implicate π' of IC_{SD}, there exists a simple prime implicate π of IC_{SS} such that $\pi \models \pi'$; thus, the proof is complete. □

Lemma 22. $\mathrm{IC}_{SA} \models \mathrm{IC}_{AD}$.

Proof. Recall that $\mathrm{IC}_{SA} = \mathrm{IC}_{SS} \wedge \mathrm{IC}_{CF}$, $\mathrm{IC}_{AD} = \mathrm{IC}_{SD} \wedge \mathrm{IC}_{CF}$. By Lemma 20, we know that $\mathrm{IC}_{SS} \models \mathrm{IC}_{SD}$. Thus, we have $\mathrm{IC}_{SA} \models \mathrm{IC}_{AD}$. □

Proposition 23. *$(\mathrm{IC}_{SA}, \mathrm{IC}_{AD})$ is simple.*

Proof. Combining Lemma 22 and Lemma 19, we obtain that for every prime implicate π' of IC_{AD}, there exists a prime implicate π of IC_{SA} such that $\pi \models \pi'$. Since IC_{CF} is a conjunction of clauses with at most two literals, we know that IC_{SA} is a conjunction of clauses with at most two literals as well, i.e., IC_{SA} is simple. Thus, with Fact 18, we obtain that for every (simple and nonsimple) prime implicate π' of IC_{AD}, there exists a simple prime implicate π of IC_{SA} such that $\pi \models \pi'$. We are done. □

Thus, according to Definition 14, for every nonsimple prime implicate π of the integrity constraint for Dung's admissibility, there exists a simple prime implicate π of the integrity constraint for solid admissibility such that $\pi \models \pi'$.

Before presenting our main results, we adapt a result from Endriss [19] to our context.

Theorem 24 (Endriss, 2018)**.** *The majority rule guarantees Γ'-feasible outcomes on Γ-rational profiles if and only if $\Gamma \models \Gamma'$ and (Γ, Γ') is simple.*

We now have all the necessary material to prove our main results.

Theorem 25. *The majority rule guarantees IC_{SD}-feasible outcomes on IC_{SS}-rational profiles.*

Proof. We obtain that $\mathrm{IC}_{SS} \models \mathrm{IC}_{SD}$ by Lemma 20 and (IC_{SS}, IC_{SD}) is simple by Proposition 21. Thus, according to Theorem 24, we have the theorem. □

Recall from Theorem 8 that no uniform quota rule preserves admissibility for all argumentation frameworks. In contrast, we have a relatively positive result when the profiles we are considering are solidly admissible sets.

Theorem 26. *The majority rule guarantees admissible outcomes on profiles of solidly admissible sets.*

Proof. This theorem is a consequence of Lemma 22, Proposition 23, and Theorem 24. □

7 Guaranteeing Admissibility with Value Restriction

As we have seen in previous sections, the majority rule fails to preserve Dung's admissibility, graded admissibility, and strong admissibility. In this section, we attempt to overcome such negative results by restricting profiles of extensions, that is, limiting profiles that the majority rule can take as input in order to obtain admissible outcomes.

Restricting the profiles that satisfy specific semantic properties is one way to restrict inputs; a more specific approach is domain restriction. In judgment aggregation, there are two well-known domain restrictions, namely, *unidimensional alignment* introduced by List [24] and *value restriction* introduced by Sen [30] for preference aggregation. In this paper, we focus on value restriction. Specifically, we make use of a result by Dietrich and List [16] from judgment aggregation and a variant from binary aggregation by Colley [13].

In this section, we reuse the model defined in Section 6. That is, we assume that there are u agents (voters), and every agent provides an extension (a set of arguments). Furthermore, we distinguish constraints that are satisfied by the agents and constraints that are supposed to be met by the collective decisions. We continue to focus on the majority rule. We first consider value restriction in the single-constraint setting; then, we turn to the binary-constraint setting.

7.1 Value Restriction With Single Constraint

We formally define the approach of value restriction to guarantee admissible outcomes during the aggregation of individual extensions. We still insist on the distinction between rationality and feasibility constraints. However, we assume that the rationality and feasibility constraints coincide. The following definition presents the single-constraint value restriction for guaranteeing admissible outcomes.

Given an extension $\Delta \subseteq Arg$ provided by an agent $i \in N$, a literal l and its corresponding argument (issue) $A \in Arg$, we say that i *agrees* with l if l is positive and $A \in \Delta$, or if l is negative and $A \notin \Delta$. We say that i *disagrees* with l if i agrees with l is not the case.

Definition 15. *Given an argumentation framework AF, a profile of extensions $\boldsymbol{\Delta} \in (2^{Arg})^u$ is value-restricted with respect to a constraint Γ if and only if for all prime implicates π of Γ containing two or more literals, there exist two distinct literals ℓ_i and ℓ_j of π such that no voter disagrees with both of them.*

This definition is a translation of the definition given by Dietrich and List [16] for judgment aggregation. Notably, our definition is similar to Definition 4.2 given by Colley [13] for binary aggregation, with a slight distinction that we have taken into consideration the constraints with prime implicates with a single literal.

Definition 15 reflects a particular kind of agreement on acceptance status of arguments among agents: given an integrity constraint Γ, for every prime implicate π of Γ, there exists a specific pair of literals in the prime implicate such that no individual disagrees with both of them. For example, suppose that there is a prime implicate $A \vee B \vee \neg C$ of an integrity constraint Γ. Without value restriction, the only thing not allowed for agents to do is rejecting A, rejecting B, and accepting C at the same time. Suppose that A and B are the pair of literals defined by Definition 15. Then, with value restriction, every voter needs to accept at least one of A and B.

Theorem 27. *Let u be odd and $\boldsymbol{\Delta}$ be a profile of extensions. If $\boldsymbol{\Delta}$ is a value-restricted profile with respect to a constraint Γ, then the majority rule guarantees Γ-feasible outcomes.*

Proof. Assume that there is a profile of extensions $\boldsymbol{\Delta}$ that is a value-restricted profile with respect to Γ. Take a prime implicate π of Γ. Without loss of generality, we suppose that π is a positive clause, which is possible because arguments are treated symmetrically by the majority rule when u is odd.

If π has two or more literals, then there are two distinct literals ℓ_i and ℓ_j of π such that no voter rejects both of them. That is, every voter accepts at least one of ℓ_i and ℓ_j. By combining this result and the assumption that u is odd, it becomes clear that at least one of ℓ_i and ℓ_j will be accepted by the majority rule. This literal will be included in $F(\boldsymbol{\Delta})$. Thus, $F(\boldsymbol{\Delta}) \models \pi$ holds.

If π has only one literal, then it is clear that every voter accepts this literal and that the outcome of the majority rule accepts it as well. Thus, $F(\boldsymbol{\Delta}) \models \pi$ also holds. $\square$

When the constraint under consideration is admissibility, i.e., $\Gamma = \mathrm{IC}_{AD}$, then a corollary of Theorem 27 for guaranteeing admissibility is straightforward.

Corollary 28. *Let u be odd and $\boldsymbol{\Delta}$ be a profile of admissible sets. If $\boldsymbol{\Delta}$ is a value-restricted profile with respect to admissibility, then the majority rule guarantees admissible outcomes.*

By comparison, we recall that Theorem 8, Theorem 13, and Example 7 have indicated that the majority rule does not preserve Dung's admissibility, graded admissibility, and strong admissibility, respectively (together with the fact that the majority rule is a special case of quota rules). The following example shows that in the case where the majority rule fails to preserve Dung's admissibility, with value restriction, we can obtain admissible outcomes by the majority rule.

Example 10. Consider $AF = \langle\{A, B, C, D\}, \{A \rightharpoonup C, B \rightharpoonup C, C \rightharpoonup D\}\rangle$, as illustrated in Figure 1. Suppose that there are three voters, each of whom proposes a set of arguments. The first voter proposes $\{A, D\}$, the second proposes $\{B, D\}$, and the third proposes $\emptyset$. All extensions are admissible, but when we apply the majority rule, we obtain an extension $\{D\}$ that is not admissible. Thus, the majority rule fails to preserve Dung's admissibility, as noted in Theorem 8.[2]

Now, let us consider another profile of extensions. Before proceeding, we recall that the integrity constraint of AF for admissibility is:

$$\mathrm{IC}_{AD} = (\neg D \vee A \vee B) \wedge \neg C \wedge \mathrm{IC}_{CF}$$

The prime implicates of IC_{AD} are $(\neg D \vee A \vee B)$, $\neg C$, and the clauses of IC_{CF} in which each of them has two negative literals. Then, for the prime implicate $(\neg D \vee A \vee B)$, we suppose that no voter disagrees with both A and B, and for the prime implicate $\neg C$, every vote agrees. In the meantime, for every prime implicate of IC_{CF}, no voter disagrees with its two literals.

Let us consider the profile of extensions in which the first agent proposes $\{A, D\}$, the second proposes $\{B, D\}$, and the third proposes $\{A, B\}$. Thus, all prime implicates are satisfied, i.e., the profile is value-restricted with respect to admissibility. By applying the majority rule, we obtain an extension $\{A, B, D\}$ that is admissible. △

The reader may have noticed that Theorem 27 assumes that the number of agents is odd. In example 10, if half of the agents vote for $\{A, D\}$ and the remaining agents vote for $\{B, D\}$, then there is a tie, which will lead to the rejection of both A and B. However, if we put a more restrictive restriction on the profile, then a similar notion of value restriction can still be used when u is even. Note that an issue is an argument, a negated issue is a negative literal.

Definition 16. *Given an argumentation framework AF, a profile of extensions $\mathbf{\Delta} \in (2^{Arg})^u$ is negatively value-restricted with respect to a constraint Γ if and only if for all prime implicates π of Γ containing two or more literals, there exist two distinct literals ℓ_i and ℓ_j of π such that no voter disagrees with both of them and at least one of them is a negated issue.*

Again, we note that this definition is similar to Definition 4.3 given by Colley [13]. Definition 16 states that for every prime implicate of the integrity constraint, there exists a particular pair of literals in the prime implicate such that no agent disagrees with both of these literals, and one of the literals is a negated issue. For example, suppose that there is a prime implicate $A \vee B \vee \neg C$ of an integrity constraint Γ. Without value restriction, the only thing not allow agents to do is rejecting A, rejecting B, and accepting B at the same time. Suppose that A and $\neg C$ are the pair of literals defined by Definition 16. Then, with negatively value restriction, every agent is required to make a choice between accepting A and rejecting B.

[2] Recall that the majority rule is a special example of a quota rule.

Theorem 29. *Let $\mathbf{\Delta}$ be a profile of extensions. If $\mathbf{\Delta}$ is a negatively value-restricted profile with respect to a constraint Γ, then the majority rule guarantees Γ-feasible outcomes.*

Proof. For the scenario where u is odd, according to Theorem 27, we know that the theorem holds.

We now consider the scenario where u is even. We assume that there is a profile of extensions $\mathbf{\Delta}$ that is negatively value-restricted with respect to Γ. For the sake of contraction, we suppose that $F(\mathbf{\Delta}) \not\models \Gamma$, i.e., the outcome of the majority rule is not Γ-feasible, which means there is a prime implicate π of Γ such that $F(\mathbf{\Delta}) \not\models \pi$.

Note that all agents are negatively value-restricted with respect to Γ. If π has two or more literals, we assume that there are two literals ℓ_i and ℓ_j of π such that no agent disagrees with both of these literals, and one of them is a negated issue. Without loss of generality, we assume that ℓ_i is a negated issue, and we do not make any assumption about whether ℓ_j is a negated issue or not.

As $F(\mathbf{\Delta}) \not\models \pi$ is the case and ℓ_i is a negated issue, there can be at most $\frac{u}{2} - 1$ agents who vote for the literal ℓ_i.[3] Then, there are at least $\frac{u}{2} + 1$ agents agreeing with ℓ_j, as the remaining agents need to vote for it. Thus, ℓ_j has received enough votes to be accepted by the majority rule. Therefore, we have that $F(\mathbf{\Delta}) \models \pi$. Thus, we have arrived at a contradiction since we have $F(\mathbf{\Delta}) \models \pi$ and $F(\mathbf{\Delta}) \not\models \pi$.

We now consider the scenario where π has only a single literal. Since $F(\mathbf{\Delta}) \not\models \pi$, the majority of the agents reject such literal, then the majority of agents reject π, which contradicts our assumption that every agent is negatively value-restricted with respect to Γ. $\square$

If the constraint we are considering is admissibility, then a corollary for guaranteeing it is a consequence of Theorem 29.

Corollary 30. *Let $\mathbf{\Delta}$ be a profile of admissible sets. If $\mathbf{\Delta}$ is a negatively value-restricted profile with respect to admissibility, then the majority rule guarantees admissible outcomes.*

In the following, we present an example to show that the majority rule guarantees admissible outcomes on a profile that is negatively value-restricted with respect to admissibility, while in the same argumentation framework, it fails to do so on a profile that is value-restricted with respect to admissibility.

Example 11. We continue to consider the $AF = \langle \{A, B, C, D\}, \{A \rightharpoonup C, B \rightharpoonup C, C \rightharpoonup D\} \rangle$ illustrated in Figure 1. Consider a profile of extension $\mathbf{\Delta} = (\{A, D\}, \{B, D\})$. The majority rule returns $\{D\}$ as the rule favouring the rejection of the arguments when there is a tie, i.e., rejecting an argument when $\frac{u}{2}$ votes against it, while the argument needs $[\frac{u}{2}+1]$ votes of support to be accepted. Even though $\mathbf{\Delta}$ is value-restricted with respect to admissibility, the majority still cannot ensure an admissible outcome when the number of agents is even.

Now, let us consider a profile of extensions that is negatively value-restricted with respect to admissibility. Recall the integrity constraint of AF for admissibility is $\mathrm{IC}_{AD} = (\neg D \vee A \vee B) \wedge \neg C \wedge \mathrm{IC}_{CF}$. The prime implicates of IC_{AD} are $(\neg D \vee A \vee B)$, $\neg C$, as well as the clauses of IC_{CF} in which each has two negative literals. Then, for the prime implicate $(\neg D \vee A \vee B)$, we suppose that no voter disagrees with both A and $\neg D$, and for the prime implicate $\neg C$, every vote agrees. Moreover, for

[3] If there are $\frac{u}{2}$ or more agents who vote for ℓ_i, then the issue (argument) will not receive enough votes for it to be accepted by the majority rule, the negated issue will be accepted, i.e., $F(\mathbf{\Delta}) \models \ell_i$. Thus, $F(\mathbf{\Delta}) \models \pi$, which contradicts our assumption that $F(\mathbf{\Delta}) \not\models \pi$.

every prime implicate of IC_{CF}, no voter disagrees with its two negative literals. We suppose that there are four voters in which the first proposes $\{A, D\}$, the second proposes $\{A\}$, the third proposes $\emptyset$, and the fourth proposes $\{B\}$. Thus, both prime implicates are satisfied, i.e., the profile is negatively value-restricted with respect to admissibility. By applying the majority rule, we obtain that none of the arguments is accepted, i.e., $\emptyset$ is the outcome of the majority rule, which is admissible. $\triangle$

7.2 Value Restriction with Rationality and Feasibility Constraints

Note that in the previous section, we have used the notions of the rationality and the feasibility constraints, but we assume such constraints are the same. In this section, we move on from the single-constraint setting to the rationality and feasibility setting for guaranteeing admissible outcomes during the aggregation of individual extensions. We first extend Definition 15 to the setting with a pair of constraints, rather than one.

Definition 17. *Given an argumentation framework AF, a profile of extensions $\mathbf{\Delta} \in (2^{Arg})^u$ is value-restricted with respect to a pair of constraints (Γ, Γ') if and only if for every prime implicate π' of Γ', there exists a prime implicate π of Γ such that $\pi \models \pi'$ holds. Furthermore, if π contains two or more literals, then there exist two distinct literals ℓ_i and ℓ_j of π such that no voter disagrees with both of them.*

If the rationality constraint implies the feasibility constraint, then according to Lemma 19, for all prime implicates of the feasibility constraint there exists a prime implicate of the rationality constraint such that the latter entails the former, which means that the first condition of Definition 17 is satisfied.

As Definition 16 has demonstrated, when an additional restriction is imposed on profiles, a similar notion of value restriction can be used when u is even.

Definition 18. *Given an argumentation framework AF, a profile of extensions $\mathbf{\Delta} \in (2^{Arg})^u$ is negatively value-restricted with respect to a pair of constraints (Γ, Γ') if and only if for every prime implicate π' of Γ', there exists a prime implicate π of Γ such that $\pi \models \pi'$ holds. Furthermore, if π contains two or more literals, then there exist two distinct literals ℓ_i and ℓ_j of π such that no voter disagrees with both of these literals and at least one of them is a negated issue.*

Next, we recreate Theorem 27 in consideration of a pair of constraints.

Theorem 31. *Let u be odd and $\mathbf{\Delta}$ be a Γ-rational profile of extensions. If $\mathbf{\Delta}$ is a value-restricted profile with respect to a pair of constraints (Γ, Γ'), then the majority rule guarantees Γ'-feasible outcomes.*

Proof. Assume that there is a profile of extensions $\mathbf{\Delta}$ that is a value-restricted profile with respect to (Γ, Γ'). We need to show that for every prime implicate π' of Γ', $F(\mathbf{\Delta}) \models \pi'$ holds. Take a prime implicate π' of Γ'. According to Definition 17, we know that there is a prime implicate π of Γ such that $\pi \models \pi'$.

Without loss of generality, we suppose that π is a positive clause; this is possible because arguments are treated symmetrically by the majority rule when u is odd. If π has two or more literals, then there are two distinct literals ℓ_i and ℓ_j of π such that no voter rejects both of them. That is, every voter accepts at least one of ℓ_i and ℓ_j. By combining this result and the assumption that u is odd, it becomes clear that at least one of ℓ_i and ℓ_j will be accepted by the majority rule. This literal will be included in $F(\mathbf{\Delta})$. Thus, $F(\mathbf{\Delta}) \models \pi$ holds. This implies that $F(\mathbf{\Delta}) \models \pi'$.

If π' has only one literal, we denote it as ℓ. By Definition 17, there is a prime implicate of Γ that contains ℓ only. Then, it is clear that every voter accepts ℓ, and the outcome of the majority rule accepts it as well. Thus, $F(\mathbf{\Delta}) \models \pi'$ also holds. $\square$

Theorem 32. *Let $\mathbf{\Delta}$ be a Γ-rational profile of extensions. If $\mathbf{\Delta}$ is a negatively value-restricted profile with respect to a pair of constraints (Γ, Γ'), then the majority rule guarantees Γ'-feasible outcomes.*

Proof. For the scenario where u is odd, according to Theorem 31, we know that the theorem holds.

We now consider the scenario where u is even. We assume that there is a profile of extensions $\mathbf{\Delta}$ that is negatively value-restricted with respect to (Γ, Γ'). For the sake of contraction, we suppose that $F(\mathbf{\Delta}) \not\models \Gamma'$, i.e., the outcome of the majority rule is not Γ'-feasible, which means that there is a prime implicate π' of Γ' such that $F(\mathbf{\Delta}) \not\models \pi'$. By Definition 18, we know that there is a prime implicate π of Γ such that $\pi \models \pi'$.

Note that all agents are negatively value-restricted with respect to (Γ, Γ'). If π has two or more literals, we assume that there are two literals ℓ_i and ℓ_j of π such that no agent disagrees with both of them, and one of them is a negated issue. Without loss of generality, we assume that ℓ_i is a negated issue, and we do not make any assumption about whether ℓ_j is a negated issue. Furthermore, as $F(\mathbf{\Delta}) \not\models \pi'$ and $\pi \models \pi'$, we have $F(\mathbf{\Delta}) \not\models \pi$.

As $F(\mathbf{\Delta}) \not\models \pi$ is the case and ℓ_i is a negated issue, there can be at most $\frac{u}{2} - 1$ agents who vote for the literal ℓ_i.[4] Then, there are least $\frac{u}{2} + 1$ agents agreeing with ℓ_j, as the remaining agents must vote for it. Thus, ℓ_j has received enough votes to be accepted by the majority rule. Therefore, we have $F(\mathbf{\Delta}) \models \pi$. Thus, we have arrived at a contradiction since we have $F(\mathbf{\Delta}) \models \pi$ and $F(\mathbf{\Delta}) \not\models \pi$.

We now consider the scenario where π has only a single literal, which we denote as ℓ. Again, since $\pi \models \pi'$ and $F(\mathbf{\Delta}) \not\models \pi'$, we have $F(\mathbf{\Delta}) \not\models \pi$, i.e., the majority of the agents reject ℓ, then the majority of agents reject π, which contradicts our assumption that every agent is negatively value-restricted with respect to Γ. $\square$

Recall that a set of arguments $\Delta \subseteq \mathit{Arg}$ is $1n$-admissible if every attacker of arguments in Δ is attacked by at least n arguments in Δ and it is conflict-free. Thus, every attacker of arguments in Δ is attacked by at least one argument in Δ, meaning that Δ satisfies admissibility in Dung's sense.[5] Note that $\mathrm{IC}_{\mathit{1nAD}}$ is the integrity constraint for $1n$-admissibility and IC_{AD} is the integrity constraint for Dung's admissibility.

Corollary 33. *Let u be odd and $\mathbf{\Delta}$ be a profile of $1n$-admissible sets. If $\mathbf{\Delta}$ is a value-restricted profile with respect to $(\mathrm{IC}_{\mathit{1nAD}}, \mathrm{IC}_{AD})$, then the majority rule guarantees admissible outcomes.*

Corollary 33 is an immediate consequence of Theorem 31 in which $\Gamma = \mathrm{IC}_{\mathit{1nAD}}$, and $\Gamma' = \mathrm{IC}_{AD}$. Notably, we know that every $1n$-admissible set of arguments is admissible in Dung's sense. Thus, the integrity constraint for $1n$-admissibility Γ implies the integrity constraint for Dung's admissibility Γ'. Thus, if for every prime implicate π of Γ' that contains two or more literals, there exist two distinct literals ℓ_i and ℓ_j of π such that no voter disagrees with both of these literals, then the majority rule guarantees admissible outcomes on profiles of $1n$-admissible sets when the number of agents u is odd. Furthermore, if one of ℓ_i and ℓ_j is a negated issue, then the requirement regarding the number of agents can even be removed.

[4] If there are $\frac{u}{2}$ or more agents who vote for the literal ℓ_i, then it will lead to a contradiction as well (as in Theorem 29).
[5] Note that n is a positive integer.

Recall that no aggregation rule preserves graded admissibility, as noted in Theorem 13. To show that Corollary 33 holds, we present an example. In this example, the majority rule guarantees admissible outcomes on a profile that is value-restricted with respect to $(\mathrm{IC}_{1nAD}, \mathrm{IC}_{AD})$.

Example 12. Consider $AF = \langle \{A, B, C, D, E, F\}, \{A \rightharpoonup E, B \rightharpoonup E, C \rightharpoonup E, D \rightharpoonup E, E \rightharpoonup F\} \rangle$, as illustrated in Figure 5.

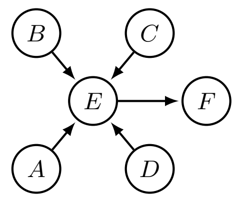

Figure 5: Scenario used in Example 12

Let us consider the property of 12-admissibility. Recall that a set of arguments $\Delta \subseteq Arg$ is 12-admissible if and only if no attacker is not attacked by 2 arguments in Δ. We first demonstrate that the majority rule will not preserve the property of 12-admissibility. To illustrate this fact, we take a profile of extensions $\boldsymbol{\Delta} = (\{F, A, B\}, \{F, C, D\}, \emptyset)$, each provided by an agent. It is easy to verify that the outcome of the majority rule is $\{F\}$, which is not admissible.

Now, we consider another profile of extensions. Before proceeding, we recall that the integrity constraint for 12-admissibility Γ of AF is: $F \rightarrow [(A \wedge B) \vee (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D) \vee (C \wedge D)] \wedge \neg E \wedge \mathrm{IC}_{CF}$, which can be rewritten as:

$$(\neg F \vee A \vee B \vee C) \wedge (\neg F \vee A \vee B \vee D) \wedge (\neg F \vee B \vee C \vee D) \wedge (\neg F \vee A \vee C \vee D) \wedge \neg E \wedge \mathrm{IC}_{CF},$$

in which $\mathrm{IC}_{CF} = (\neg A \vee \neg E) \wedge (\neg B \vee \neg E) \wedge (\neg C \vee \neg E) \wedge (\neg D \vee \neg E) \wedge (\neg E \vee \neg F)$, a conjunction of clauses containing two negative literals. The integrity constraint for Dung's admissibility Γ' of AF is:

$$(\neg F \vee A \vee B \vee C \vee D) \wedge \neg E \wedge \mathrm{IC}_{CF}.$$

Clearly, for every prime implicate π' of Γ', there is a prime implicate π of Γ such that $\pi \models \pi'$. Furthermore, we suppose that for the prime implicate $(\neg F \vee A \vee B \vee C)$ of Γ, no voter disagrees with A and B, for $(\neg F \vee A \vee B \vee D)$, no voter disagrees with A and B, for $(\neg F \vee B \vee C \vee D)$, no voter disagrees with B and C, and for $(\neg F \vee A \vee C \vee D)$, no voter disagrees with A and C. Now, we assume that there are three agents: the first agent proposes $\{F, A, B\}$, the second proposes $\{F, C, D\}$, and the third proposes $\{C\}$. Thus, for every prime implicate π' of Γ', there exists a prime implicate π of Γ such that $\pi \models \pi'$ holds, and either there exist two distinct literals ℓ_i and ℓ_j of π such that no voter disagrees with both of them or π has only one literal, i.e., $\boldsymbol{\Delta}$ is value-restricted with respect to (Γ, Γ'). By applying the majority rule, we obtain an extension $\{C\}$ that is admissible, as expected. △

Corollary 34. *Let $\boldsymbol{\Delta}$ be a profile of $1n$-admissible sets. If $\boldsymbol{\Delta}$ is a negatively value-restricted profile with respect to $(\mathrm{IC}_{1nAD}, \mathrm{IC}_{AD})$, then the majority rule guarantees admissible outcomes.*

Corollary 34 is a consequence of Theorem 32, in which $\Gamma = \mathrm{IC}_{1nAD}$ (which represents the integrity constraint for $1n$-admissibility), and $\Gamma' = \mathrm{IC}_{AD}$ (which represents the feasibility constraint for Dung's admissibility). Now, consider an example for which for every prime implicate of Γ, for a pair of literals for which every agent will not reject both of them, at least one is negative. Once again, the majority rule is well-behaved.

Example 13. Again, we consider the $AF = \langle\{A, B, C, D, E, F\}, \{A \rightharpoonup E, B \rightharpoonup E, C \rightharpoonup E, D \rightharpoonup E, E \rightharpoonup F\}\rangle$ illustrated in Figure 5, and let Γ be 12-admissibility and let Γ' be Dung's admissibility. Recall that the integrity constraint for 12-admissibility of AF is $\Gamma = (\neg F \vee A \vee B \vee C) \wedge (\neg F \vee A \vee B \vee D) \wedge (\neg F \vee B \vee C \vee D) \wedge (\neg F \vee A \vee C \vee D) \wedge \neg E \wedge \mathrm{IC}_{CF}$ and that the integrity constraint for Dung's admissibility of AF is $\Gamma' = (\neg F \vee A \vee B \vee C \vee D) \wedge \neg E \wedge \mathrm{IC}_{CF}$.

We suppose that for the prime implicate $(\neg F \vee A \vee B \vee C)$ of Γ, no voter disagrees with A and $\neg F$, for $(\neg F \vee A \vee B \vee D)$, no voter disagrees with A and $\neg F$, for $(\neg F \vee B \vee C \vee D)$, no voter disagrees with B and $\neg F$, and for $(\neg F \vee A \vee C \vee D)$, no voter disagrees with A and $\neg F$. For the prime implicate $\neg E$, every vote agrees with it, and for every prime implicate of IC_{CF}, no voter disagrees with its two negative literals.

Consider a profile of extensions $\boldsymbol{\Delta}$, in which the first agent proposes $\{F, A, B\}$, the second proposes $\{A\}$, the third proposes $\{B\}$, and the fourth proposes $\emptyset$. Thus, for every prime implicate π' of Γ', there exists a prime implicate π of Γ such that $\pi \models \pi'$ holds, and either there exist two distinct literals ℓ_i and ℓ_j of π such that no voter disagrees with both of these literals and at least one of them is a negated issue or π has only one literal, i.e., $\boldsymbol{\Delta}$ is negatively value-restricted with respect to (Γ, Γ'). By applying the majority rule, we obtain an extension $\emptyset$ that is admissible. $\triangle$

8 Conclusion

In this paper, we have explored the possibility of obtaining admissible sets of arguments during the aggregation of extensions of abstract argumentation frameworks. We have demonstrated that the majority rule is not well-behaved during the aggregation of Dung's admissibility, graded admissibility, or strong admissibility. To overcome such negative results, we have made use of the concept of solid admissibility, which allows for strong assignments of status to arguments. We have proposed a model that allows for a clear distinction between rationality and feasibility constraints, which are supposed to be satisfied by individual decisions and collective decisions, respectively. We have shown the majority rule, a fair rule that is appealing on normative grounds, guarantees admissible sets on solidly admissible sets by making use of our new model. We further show that if a profile is value-restricted in the sense that only extensions with some specific characteristics are allowed to be the inputs of the aggregation rule, then the majority rule guarantees the admissibility of the extensions.

For future work, we first note that there are other notions of admissibility in the literature (such as weak admissibility [4]) that deserve further study. Second, even though admissibility is a fundamental property of extensions of argumentation frameworks, other properties are of particular interest as well. Thus, it would be interesting to study the preservation of other semantic properties by making use of our new model. Third, more application scenarios of our model with rationality and feasibility constraints should be investigated. For example, Corollary 34 states that if a rationality constraint entails a feasibility constraint, then it is possible to guarantee feasible outcomes on rational profiles if the requirement is satisfied. This characteristics indicates that there is considerable work that can be done in the future, as logical entailment relations are ubiquitous in argumentation frameworks.

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