Collective Argumentation with Topological Restrictions

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Abstract. Collective argumentation studies how to reach a collective decision that is acceptable to the group in a debate. I introduce the concept of topological restriction to enrich collective argumentation. Topological restrictions are rational constraints assumed to be satisfied by individual agents. We assume that in a debate, for every pair of arguments that are being considered, every agent indicates whether the first one attacks the second, i.e., an agent's argumentative stance is characterized as an argumentation framework, and only argumentation frameworks that satisfy topological constraints are allowed. The topological constraints we consider in this paper include acyclicity, symmetry, as well as a newly defined topological property called t-self-defense. We show that when the profile of argumentation frameworks provided by agents satisfies topological restrictions, impossibility results during aggregation can be avoided. Furthermore, if a profile is topological-restricted with respect to t-self-defense, then the majority rule preserves admissibility during aggregation.

Keywords: Collective Argumentation · Topological Restriction · Social Choice Theory.

1 Introduction

Abstract argumentation theory is a formalism that deals with the formalization of argumentation. It has been applied for over twenty years to analyze the argument justification. When there are several agents involved in a debate, such as juridical and parliamentary debates, they may have different opinions on the evaluation of the acceptability of arguments or the justification of attacks between arguments. Collective argumentation has been discussed extensively in the literature of formal argumentation (see [7, 6]). Among them, some are dedicated to investigating the aggregation of arguments [13, 23, 12, 9], while others study the aggregation of attacks [8, 21, 7, 11, 10].

The problem of aggregation of abstract argumentation frameworks has received attention in the literature in the last decade or so [13, 23, 12, 7]. On the methodology level, some study the performance of simple and straightforward rules, such as the majority rule, quota rules, while some other study rules with high complexity, such as distance-based rules. It is worth mentioning that while different aggregation mechanisms, different aggregation entities have been employed, a common feature of these work is that they study the aggregation of argumentation frameworks without constraints. In other words, no restriction is imposed on the argumentation frameworks. Each individual agent provides an arbitrary argumentation framework that represents her argumentative stance in a debate. In this case, we assume that for every pair of arguments that are being considered in a debate, every agent indicates whether the first attack the second. Given a semantic property agreed upon by the individual agents, the output may or may not satisfy such property.

I propose the notion of topological restriction to enrich collective argumentation. Topological restrictions will help us to get rid of argumentation frameworks that are not desirable. For example, we may consider it irrational for an individual agent to support argumentation frameworks that contain odd-length cycles, in which an argument may indirectly attack and support another. In this case, the acceptance status of the second argument is controversial and we would like to avoid such controversy. In this case, we can require that agents' argumentation frameworks satisfy acyclicity. For acyclic argumentation frameworks, the acceptance status of arguments is unambiguous as the grounded extension coincides with the unique preferred extension that is also stable. There are other topological properties that can help us avoid controversy, such as symmetry. For symmetric argumentation frameworks, the attack-relation is symmetric. As a consequence, every symmetric argumentation framework is coherent (which means that every preferred extension is stable) and relatively grounded (which means that the grounded extension is the intersection of all its preferred extensions).

Our contribution is two-fold: first, we introduce the notion of topological restriction to the aggregation of argumentation frameworks and study several topological restrictions during aggregation, including acyclicity, symmetry, and a newly defined topological property called t-self-defense. We show that with topological restrictions, impossibility results during aggregation of attack-relations can be avoided. To be specific, there are some aggregation rules that preserve demanding properties. Also, we show that, if a profile is topological-restricted with respect to t-self-defense, then the majority rule, a rule that is very appealing on normative grounds, as it treats all agents in a "fair" manner, preserves admissibility during aggregation.

The remainder of the paper is organized as follows. Section 2 presents relevant concepts from the theory of abstract argumentation, including some of the fundamentals of the model of abstract argumentation, topological property, and semantics agreement. Section 3 introduces our model and Section 4 introduces the concept of topological restriction. Section 5 presents our preservation results with topological restrictions of acyclicity and symmetry. Section 6 introduces a topological property called t-self-defense and preservation results for admissibility with t-self-defense. Section 7 concludes the paper.

2 Argumentation framework and topological property

An argumentation framework is a pair $AF = \langle Arg, \rightharpoonup \rangle$, in which Arg is a set of arguments and $\rightharpoonup$ is a set of binary relations called the attack relation built on Arg. Given two arguments $A, B \in Arg$, $A \rightharpoonup B$ if and only if A attacks B. Given a set of arguments $\Delta \subseteq Arg$, we say that Δ is *conflict-free* if there are no arguments $A, B \in Arg$ such that $A \rightharpoonup B$ is the case; we say that Δ defends $A \in Arg$ if for every argument $B \in Arg$ with $B \rightharpoonup A$ is the case, there is an argument $C \in \Delta$ such that $C \rightharpoonup B$; we say that Δ is self-defending if Δ defends every argument in Δ; we say that Δ is *admissible* if Δ is conflict-free and self-defending; furthermore, we say that:

- Δ is complete if Δ is admissible and every argument defended by Δ is included in Δ.
- Δ is grounded if Δ is the minimal complete extension (w.r.t. set inclusion)
- Δ is preferred if Δ is a maximal admissible set (w.r.t. set inclusion)
- Δ is stable if Δ is conflict-free and attacks every argument that is not in Δ.

A semantics defines which set of arguments can be accepted, which can be considered as a property of sets of arguments. We now present another family of properties considered in the literature in abstract argumentation, namely *topological properties* of argumentation frameworks. While topological properties of argumentation frameworks have no immediately apparent relationships with argumentation semantics, they play an important role in the study of such semantics. As early as in the seminal paper by Dung [16], well-foundedness has been identified as a topological properties and has been shown that it is a sufficient condition for agreement among grounded, preferred, and stable semantics, namely the grounded extension is the only preferred and stable extension.

Definition 1. *An argumentation framework is well-founded if and only if there exists no infinite sequence $A_0, A_1, \cdots, A_n$ of arguments such that for each i, $A_{i+1} \rightharpoonup A_i$ is the case.*

In the case of a finite argumentation framework, well-foundedness coincides with *acyclicity* of the attack relation.

Definition 2. *An argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$ is coherent if every preferred extension of AF is stable.*

The absence of odd-length cycles is a sufficient condition to ensure that the argumentation framework is coherent, i.e., ensure that stable extensions exist and coincide with preferred extensions.

Definition 3. *An argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$ is a symmetric argumentation framework if $\rightharpoonup$ is symmetric, nonempty and irreflexive.*

In other words, an argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$ is symmetric if for any pair of argument $A, B \in Arg$ with A attacks B is the case, then B will be counter-attacked by A.

Other topological properties of argumentation frameworks in the literature include *antisymmetry* (i.e., the absence of mutual attack between arguments), *directionality property*, introduced in [4], *SCC-recursiveness property*, introduced in [5], *almost determinedness property*, introduced in [3], as well as *limited controversy* introduced by Dung in his seminal work [17].

3 The aggregation model

Fix a set of arguments Arg as well as a set of agents $N = \{1, \cdots, n\}$, suppose that each agent provides an argumentation framework, reflecting her individual views on the status of possible attacks between arguments. Thus, we are given a profile of attack-relations $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$. Sometimes we may want to aggregate individual argumentation frameworks to obtain a single argumentation framework that reflects the consensus of the group, what would be a good method to arrive at this goal? In this paper, we focus on the method from social choice theory, an aggregation rule is a function that maps any given profile of attack-relations into a single attack-relation $F : (2^{Arg \times Arg})^n \rightarrow 2^{Arg \times Arg}$. We use $N^{\boldsymbol{\rightharpoonup}}_{att} := \{i \in N \mid att \in (\rightharpoonup_i)\}$ to denote the set of *supporters* of the attack *att* in profile $\boldsymbol{\rightharpoonup}$.

Now we present several intuitively desirable property of aggregation rules. Such properties are called axioms in the literature on social choice theory [1]. All of these axioms are adapted of axioms formulated in the literature on graph aggregation [18] and have been defined in the work by Chen and Endriss [12].

Definition 4. *An aggregation rule is said to be* neutral *if* $N^{\boldsymbol{\rightharpoonup}}_{att} = N^{\boldsymbol{\rightharpoonup}}_{att'}$ *implies* $att \in F(\boldsymbol{\rightharpoonup}) \Leftrightarrow att' \in F(\boldsymbol{\rightharpoonup})$ *for all profiles* $\boldsymbol{\rightharpoonup}$ *and all attacks* att, att'.

Definition 5. *An aggregation rule is said to be* independent *if* $N^{\boldsymbol{\rightharpoonup}}_{att} = N^{\boldsymbol{\rightharpoonup}'}_{att}$ *implies* $att \in F(\boldsymbol{\rightharpoonup}) \Leftrightarrow att \in F(\boldsymbol{\rightharpoonup})$ *for all attacks att and all profiles* $\boldsymbol{\rightharpoonup}, \boldsymbol{\rightharpoonup}'$.

Definition 6. *An aggregation rule is said to be* unanimous *if* $F(\boldsymbol{\rightharpoonup}) \supseteq (\rightharpoonup_1) \cap \cdots \cap (\rightharpoonup_n)$ *is the case for all profiles* $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$.

Definition 7. *An aggregation rule is said to be* grounded *if* $F(\boldsymbol{\rightharpoonup}) \subseteq (\rightharpoonup_1) \cup \cdots \cup (\rightharpoonup_n)$ *is the case for all profiles* $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$. .

Thus, an aggregation rule is neutral if two attacks receive the same votes in a profile, then the acceptance status of them are the same in the outcome, i.e., attacks are treated symmetrically; an aggregation rule is independent if the acceptance of an attack only depends on its supporters; unanimity assumes that if an attack is accepted by everyone, then it should be accepted in the collective outcome; groundedness postulates that only attacks with at least one supporter can be collective accepted.

Two special families of aggregation we consider in this paper are the quota rules and the dictatorship rules. All of them are simple rules that are adaptations from other parts of social choice theory, such as judgment aggregation [19] and graph aggregation [18]. Notably, all of them are well defined in [12].

Definition 8. *Let $q \in \{1, \ldots, n\}$. The **quota rule** F_q with quota q accepts all those attacks that are supported by at least q agents:*

$$F_q(\boldsymbol{\rightharpoonup}) = \{att \in Arg \times Arg \mid \#N^{\boldsymbol{\rightharpoonup}}_{att} \geqslant q\}$$

The *majority rule* is the quota rule F_q with $q = \lceil \frac{n+1}{2} \rceil$. Two further quota rules are also of special interest. The *unanimity rule* only accepts attacks that are supported by everyone, i.e., this is F_q with $q = n$. The *nomination rule* is the quota rule F_q with $q = 1$. Despite being a somewhat extreme choice, the nomination rule has some intuitive appeal in the context of argumentation, as it reflects the idea that we should take seriously any conflict between arguments raised by at least one member of the group.

Definition 9. *The **dictatorship rule** F_{D_i} of dictator $i \in N$ accepts all those attacks that are accepted by agent i:*

$$F_{D_i}(\boldsymbol{\rightharpoonup}) = \rightharpoonup_i$$

Thus, under a dictatorship, to compute the outcome, we simply copy the attack-relation of the dictator. Intuitively speaking, dictatorships in particular, are unattractive rules, as they unfairly exclude everyone except i from the decision process.

We consider the preservation of semantic properties of argumentation frameworks. An AF-property $P \subseteq 2^{Arg \times Arg}$ is the set of all attack-relations on Arg that satisfy P, we denote it by $P(\rightharpoonup)$. For example, non-emptiness of the grounded extension is a simple semantic property, an AF satisfies such property if there is at least one argument that is not attacked by any argument in AF.

Definition 10 (Preservation). *Fix a finite set Arg of arguments and a set of $N = \{1, \cdots, n\}$ agents. Suppose that each agent provides an argumentation framework, which reflects her individual views on the status of possible attacks between arguments. An aggregation rule F is said to preserve a property P if for every profile $\boldsymbol{\rightharpoonup}$ it is the case that $P(\rightharpoonup_i)$ being the case for all agents $i \in N$ then $P(F(\boldsymbol{\rightharpoonup}))$.*

Thus, in the case where all agents' attack-relations satisfy P, F preserves P if the outcome of F satisfies P as well. The AF-properties we will discuss in this paper include *conflict-freeness*, *admissibility*, *being an extension under a specific semantics*, *non-emptiness of the grounded extension*, and *coherence*. *Conflict-freeness* is a AF-property which requires that, if for all sets $\Delta \subseteq Arg$, whenever Δ is conflict-free in $\langle Arg, \rightharpoonup_i \rangle$ for all agents $i \in N$, we would like that Δ is conflict-free in $\langle Arg, F(\boldsymbol{\rightharpoonup}) \rangle$. If it is the case, then we say that F preserves conflict-freeness. The AF-property of *admissibility* can be defined in the same way. *Being an extension under a specific semantics* require that, given a set of arguments Δ, Δ is an extension of a given semantics in $\langle Arg, \rightharpoonup_i \rangle$ for all agents $i \in N$, then Δ is also an extension of the semantics of $\langle Arg, F(\boldsymbol{\rightharpoonup}) \rangle$. Finally, *coherence* is also an attractive properties, because—if satisfied by an argumentation framework—they ensure that preferred and stable extensions will coincide and result in the same recommendations about which arguments to accept, thereby making decisions less controversial. It is worth noting that topological properties are a special subset of AF-properties.

4 Topological restriction

In this section, I introduce the notion of topological restriction for the aggregation of attack-relations of argumentation frameworks. What are the intuitions behind this notion? First, while it is easy to verify that most semantic properties cannot be preserved by the majority rule[1], we cannot get things going for any aggregation rule that satisfies desirable axiomatic requirements. As an example, we present the following impossibility theorem.

Theorem 1 (Chen and Endriss, 2019). *For $|Arg| \geqslant 5$, any unanimous, grounded, and independent aggregation rule F that preserves either complete or preferred extensions must be a dictatorship.*

To prove Theorem 1, Chen and Endriss have used a technique developed by Endriss and Grandi for the more general framework of graph aggregation, which in turn was inspired by the seminal work on preference aggregation of Arrow [2]. Clearly, Theorem 1 is an impossibility result. At the heart of Theorem 1 (as well as other impossibility results), there are three types of conditions: axioms of aggregation rules, semantic properties of argumentation frameworks, as well as argumentation frameworks allowed to input. To cope with such negative results, one direction is relaxing such conditions, requirements, or argumentation frameworks allowed.

Before going any further, we recall approaches that aim to deal with impossibility results in the literature on social choice theory. In the literature on judgment aggregation, there is an approach that proposes to restrict the range of agendas, namely restricting the range of agendas on which we can perform satisfactorily with aggregation rules. Another approach in judgment aggregation is *domain restriction*, namely restricting the profiles allowed to input. Introduced by List [20], *unidimensional alignment* is a widely known way of domain restriction. The idea of unidimensional alignment is that only profiles which are unidimensionally aligned are allowed to the aggregation rule. *Value restriction* is another type of domain restriction, for which the idea was first introduced by Sen [22] for preference aggregation and later generalized by Dietrich and List [15] for judgment aggregation. They show that if a profile is *value-restricted* in the sense that for every minimal inconsistent subset X of the agenda, there exists two formulas $\varphi, \psi \in X$ such that no agent accepts both φ and ψ, then the outcome of the majority rule will be consistent (meaning that no p and $\neg p$ get accepted at the same time).

Recently, Chen considers value-restriction during the aggregation of extensions of AFs [10]. He assumes that individual agents choose different extensions

[1] A notable exception is conflict-freeness, which can be preserved by the majority rule [12].

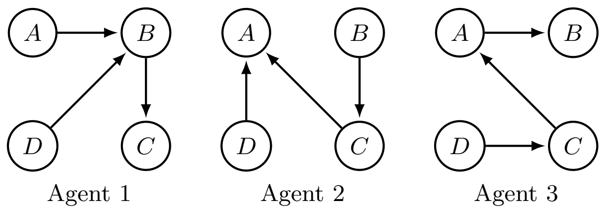

Fig. 1. Example for a profile with $Arg = \{A, B, C, D\}$.

when confronted with the same abstract argumentation framework and study the preservation of properties of extensions. Chen uses a formula Γ to describe such a property of extensions, and refers to Γ as an integrity constraint. He shows that if for every prime implicates π of the integrity constraint Γ of a given semantic properties, there exists two distinct literals such that no agent rejects both, then the majority rule preserves admissible outcomes [10].

I propose to restrict the input of the aggregation rule in the sense that only argumentation frameworks with the specific feature are allowed to the aggregation rule. In the work by Chen and Endriss in which the model is the one we adopt in this paper, there is no restriction made to the argumentation frameworks put forward by individual agents. While there are many argumentation frameworks that contain undesirable features, it is very natural to restrict the inputs to the family of argumentation frameworks without such features.

Definition 11. *A profile* $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$ *is* topological-restricted *with respect to a constraint* Γ *if and only if* $\rightharpoonup_i$ *satisfies* Γ *for all* $i \in N$.

Thus, given a constraint Γ which is a topological property of argumentation frameworks, a profile is topological-restricted with respect to Γ if every individual argumentation framework satisfies Γ. When we perform aggregation on the profile, only argumentation frameworks satisfying Γ are allowed to aggregation rules. While most preservation results of demanding properties are negative [12], possible results may be obtained when restrictions are imposed. Consider the following example:

Example 1. Let us consider an example that illustrates the preservation of acyclicity with majority. Recall that the majority includes an attack if and only if a majority of the individual agents do. Consider three agents for which the first one supports $A \rightharpoonup B$ and $B \rightharpoonup C$, the second supports $B \rightharpoonup C$ and $C \rightharpoonup A$, and the third supports $C \rightharpoonup A$ and $A \rightharpoonup B$. Clearly, every individual argumentation framework in this profile satisfies acyclicity. If we apply this rule to the profile shown in Figure 1, then we obtain the argumentation framework that contains three attacks $A \rightharpoonup B$, $B \rightharpoonup C$, and $C \rightharpoonup A$, which forms a cycle, violating acyclicity. But if no individual agent supports $A \rightharpoonup B$, for example, acyclicity will be preserved in this case. Thus, we can think that rejecting $A \rightharpoonup B$ is a topological restriction Γ. If a profile is topological-restricted with respect to Γ, the majority rule preserves acyclicity in this specific case.

Example 1 cares only about a specific profile. We now present a proposition which is more concrete, showing that if a profile is topological-restricted with respect to a constraint Γ, then every plausible aggregation rule preserves an AF-property. The AF-property is the nonemptiness of the grounded extension. Here we recall the work by Chen and Endriss, who present a preservation result for nonemptiness of the grounded extension.

Theorem 2 (Chen and Endriss, 2019). *If $|Arg| \geqslant n$, then under any neutral and independent aggregation rule F that preserves nonemptiness of the grounded extension at least one agent must have veto powers.*

Proposition 1. *Let Γ be a topological property that requires that there is an argument $A \in Arg$ that is unattacked in $\rightharpoonup_i$ for all $i \in N$. Given a profile $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$ which is topological-restricted with respect to Γ, then every aggregation rule that is grounded preserves the nonemptiness of the grounded extension.*

Proof. Let F be the aggregation rule that is grounded. Consider a profile of attack-relations $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$. Suppose that $A \in Arg$ is an unattacked argument in $\rightharpoonup_i$ for all $i \in N$. Clearly, as F is grounded, i.e., $F(\boldsymbol{\rightharpoonup}) \subseteq \rightharpoonup_1 \cup, \cdots, \cup \rightharpoonup_n$, no argument attacks A in $F(\boldsymbol{\rightharpoonup})$.

Thus, a positive result is obtained when the profile is topological-restricted with respect to a topological property that is weak and easy to satisfy. Proposition 1 provides a clue on how to overcome negative results during aggregation. In the following section, we study more topological restrictions, including notable topological properties in the literature, such as acyclicity, symmetry, as well as t-self-defense, a newly defined topological property, and we are going to show that the majority rule is well behaved with it.

5 Preservation results with topological restrictions

In this section, we present preservation results for AF-properties with topological restrictions. The topological restrictions include acyclicity and symmetry. Most of our results have the following form: there is an aggregation rule preserving an AF-property, and the AF-property coincides with the second AF-property, if a profile of argumentation frameworks whose members satisfy a topological restriction, then the preservation result for one semantics can be extended to another.

5.1 Acyclicity

Acyclicity is an important property of argumentation frameworks. As we have mentioned in previous sections, if an argumentation framework is acyclic, then it contains a single extension which is the only complete, preferred and stable extension.

Definition 12. *A profile* $\rightharpoonup = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$ *is topological-restricted with respect to acyclicity if* $\rightharpoonup_i$ *is acyclic for all* $i \in N$*.*

Thus, a profile is *topological-restricted* with respect to acyclicity if every argumentation framework in the profile satisfies acyclicity.

Fact 3 *In the case of a finite argumentation framework, well-foundedness coincides with acyclicity of the attack relation.*

Theorem 4 (Dung, 1995). *Every acyclic argumentation framework has exactly one complete extension which is grounded, preferred and stable.*

Proposition 2 (Chen and Endriss, 2019). *The nomination rule preserves stable extensions.*

Fact 5 *Every stable extension is preferred and complete.*

We now present a preservation results for preferred and complete extensions with topological restrictions. The preservation of both AF-properties has been discussed in-depth by Chen and Endriss in [12], who show that the preservation of extensions of either preferred or complete semantics is impossible by means of a "simple" aggregation rule (a rule that satisfies three "fair" axioms), unless the rule in use is a dictatorship.

Theorem 6 (Chen and Endriss, 2019). *For* $|Arg| \geqslant 5$*, any unanimous, grounded, and independent aggregation rule* F *that preserves either preferred or complete extensions must be a dictatorship.*

Proposition 3. *For any profile of attack-relations* $\rightharpoonup = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$*, if* $\rightharpoonup$ *is topological-restricted with respect to acyclicity, then the nomination rule preserves preferred and complete extensions.*

Proof. Let F be the nomination rule. Suppose that $\Delta \subseteq Arg$ be the set of arguments that is preferred or complete in $\rightharpoonup_i$ for all $i \in N$. According to Theorem 4, Δ is stable in $\rightharpoonup_i$ for all $i \in N$. Thus, as F preserves stable extensions, Δ is stable in $F(\rightharpoonup)$. By the fact that every stable extension is preferred and complete, we get that Δ is preferred or complete in $F(\rightharpoonup)$, we are done.

5.2 Symmetry

In this section, we consider the topological restriction of symmetry.

Definition 13. *An argumentation framework* $AF = \langle Arg, \rightharpoonup \rangle$ *is a symmetric argumentation framework if* $\rightharpoonup$ *is symmetric, nonempty and irreflexive.*

Before going any further, we present a result regarding the preservation of conflict-freeness in [12], which shows that every plausible aggregation rule preserves it.

Theorem 7 (Chen and Endriss, 2019). *Every aggregation rule F that is grounded preserves conflict-freeness.*

We also present a result concerning the relation between admissibility and conflict-freeness in [14], which shows that admissible sets and conflict-free sets coincide in symmetric argumentation frameworks.

Proposition 4 (Coste-Marquis et al., 2005). *Let $AF = \langle Arg, \rightharpoonup \rangle$ be a symmetric argumentation framework, a set of arguments $\Delta \in Arg$ is admissible if and only if it is conflict-free.*

Definition 14. *A profile $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$ is topological-restricted with respect to symmetry if $\rightharpoonup_i$ is symmetric for all $i \in N$.*

With Theorem 7 and Proposition 4, we are ready to present a preservation result for admissibility with topological restrictions.

Theorem 8. *For any profile of attack-relations $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$, if $\boldsymbol{\rightharpoonup}$ is topological-restricted with respect to symmetry, then every aggregation rule that is grounded and neutral preserves admissibility.*

Proof. Consider a profile of attack-relations $\boldsymbol{\rightharpoonup} = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$. Let F be an aggregation rule that is grounded and neutral. Let $\Delta \subseteq Arg$ be a set of arguments that is admissible in $\rightharpoonup_i$ for all $i \in N$. Clearly, Δ is conflict-free $\rightharpoonup_i$ for all $i \in N$. As F preserves conflict-freeness (cf. Theorem 7), we get that Δ is conflict-free in $F(\boldsymbol{\rightharpoonup})$. According to neutrality of F and the fact that the profile is topological-restricted with respect to symmetry, for every pair of arguments $A, B \in Arg$, $A \rightharpoonup B$ and $B \rightharpoonup A$ are treated symmetrically, and they receive the same votes, i.e., if $A \rightharpoonup B$ get accepted by F, so does $B \rightharpoonup A$. Thus, $F(\boldsymbol{\rightharpoonup})$ is symmetric. Combining with Proposition 4, we get that Δ is admissible in $F(\boldsymbol{\rightharpoonup})$, we are done.

Proposition 5 (Coste-Marquis et al., 2005). *Every symmetric argumentation framework is coherent.*

Recall that coherence is a property that ensures that the stable and the preferred semantics coincide. It is defined as the AF-property of every preferred extension being a stable extension. We say that an aggregation rule F preserves coherence if it is the case that, whenever $\langle Arg, \rightharpoonup_i \rangle$ is coherent for all $i \in N$, then $F(\boldsymbol{\rightharpoonup})$ is coherent. Chen and Endriss [12] have shown that preservation of coherence is impossible unless we use dictatorships.

Theorem 9 (Chen and Endriss, 2019). *For $|Arg| \geqslant 4$, any unanimous, grounded, and independent aggregation rule F that preserves coherence must be a dictatorship.*

When the profile under consideration is topological-restricted with respect to symmetry, the impossibility result can be avoided.

Proposition 6. *For any profile of attack-relations $\rightharpoonup = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$, if $\rightharpoonup$ is topological-restricted with respect to symmetry, then any aggregation rule that is grounded and neutral preserves coherence.*

Proof. Let F be an aggregation rule that is grounded and neutral. Consider a pair of arguments $A, B \in Arg$ as well as the attacks $A \rightharpoonup B$, $B \rightharpoonup A$ between them. According to the fact that $\rightharpoonup$ is a profile that is topological-restricted with respect to symmetry and the fact that F is an aggregation rule that is grounded and neutral, we get that $A \rightharpoonup B$ and $B \rightharpoonup A$ receive the same votes and they are treated symmetrically by F. Thus, if $A \rightharpoonup B$ get accepted, then $B \rightharpoonup A$ get accepted as well. As a consequence, $F(\rightharpoonup)$ is a symmetric argumentation framework. Together with Proposition 5, we get that $F(\rightharpoonup)$ is coherent.

Recall that Theorem 1 has shown that only dictatorships preserve preferred extensions. Interestingly, with the topological restriction of symmetry, we obtain a much more positive result.

Theorem 10. *For any profile of attack-relations $\rightharpoonup = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$, if $\rightharpoonup$ is topological-restricted with respect to symmetry, then the nomination rule preserves preferred extensions.*

Proof. Let F be the nomination rule. Suppose that $\Delta \subseteq Arg$ is a set of arguments that is preferred in $\rightharpoonup_i$ for all $i \in N$. According to Proposition 5, Δ is stable in $\rightharpoonup_i$ for all $i \in N$. Thus, as F preserves stable extensions (cf. Proposition 2), Δ is stable in $F(\rightharpoonup)$. By the fact that every stable extension is preferred, we get that Δ is preferred in $F(\rightharpoonup)$, we are done.

6 The majority rule and topological constraints

In this section, we focus on the preservation of semantic properties with topological constraints. The aggregation rule we pay particular attention to is the majority rule. We first show that the majority rule does not preserve admissibility, a property at the heart of all classical semantics. Then, we define a topological property, followed by a result that shows that if a profile of attack-relations is topological-restricted with respect to the property, then the majority rule preserves admissibility during aggregation.

Example 2. Consider the profile illustrated in Figure 2, $\{A_1, A_2, A_3, C\}$ is admissible in every individual's argumentation framework, but it is not admissible in the outcome of the majority rule. Thus, the majority rule does not preserve admissibility.

Next, we introduce the notion of *the union of attack-relations* of profiles of attack-relations.

Definition 15. *Given a profile $\rightharpoonup = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$, we denote the union of attack-relations of $\rightharpoonup$ by $\rightharpoonup_u$, i.e., $\rightharpoonup_u = \rightharpoonup_i \cup \cdots \cup \rightharpoonup_n$.*

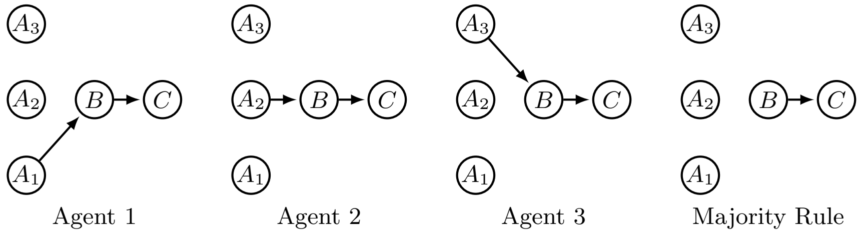

Fig. 2. Scenarios used in Example 2.

In other words, the union of attack-relations of a profile if it includes those attacks that accepted by at least one agent. For instance, in Example 2, $\rightharpoonup_u = \{A_1 \rightharpoonup B, A_2 \rightharpoonup B, A_3 \rightharpoonup B, B \rightharpoonup C\}$.

Definition 16. *Given a profile of attack-relations $\rightharpoonup = (\rightharpoonup_1, \ldots, \rightharpoonup_n)$. We say that $\rightharpoonup$ is topological-restricted with respect to* t-self-defense *if for every attack $B \rightharpoonup C \in \rightharpoonup_u$ whose attacker B has two or more attackers in $\langle Arg, \rightharpoonup_u \rangle$, for every pair of attackers A_i, A_j of B no agent rejects both $A_i \rightharpoonup B$ and $A_j \rightharpoonup B$.*

In other words, for every attack $att = B \rightharpoonup C \in \rightharpoonup_u$, we denote the attackers of B by $A_1, \cdots, A_k$ with $k \geqslant 2$, i.e., $A_1 \rightharpoonup B, \cdots, A_k \rightharpoonup B \in \rightharpoonup_u$, there are at least two attackers A_i and A_j of B for which no agent rejects both $A_i \rightharpoonup B$ and $A_j \rightharpoonup B$.

Theorem 11. *If the number of agents is odd, then for any profile of attack-relations $\rightharpoonup = (\rightharpoonup_1, \cdots, \rightharpoonup_n)$, if $\rightharpoonup$ is a profile that is topological-restricted with respect to t-self-defense, then the majority rule preserves admissibility.*

Proof. Assume that $\Delta \subseteq Arg$ is admissible in $\rightharpoonup_i$ for all $i \in N$. Let F be the majority rule. According to Theorem 7, Δ is conflict-free in $F(\rightharpoonup)$. It remains to show that Δ is self-defending in $F(\rightharpoonup)$. To arrive at this goal, we need to show that for every argument $C \in \Delta$, if C is attacked by some argument B, then B is attacked by some argument in Δ in the outcome of the majority rule.

Suppose that $B \rightharpoonup C \in F(\rightharpoonup)$ is the case, then $B \rightharpoonup C \in \rightharpoonup_u$. If B has only one attacker in $\langle Arg, \rightharpoonup_u \rangle$, and we denote it by A, then any agent who supports $B \rightharpoonup C$ would be required to support $A \rightharpoonup B$, meaning that the majority of agents support $A \rightharpoonup B$. Thus, in this scenario, $B \rightharpoonup C$ and $A \rightharpoonup B$ receive the same votes, which is also a majority of supports from agents. If $A \notin \Delta$, then Δ is not self-defending in such agents' argumentation frameworks, contradicting our earlier assumption. Thus, $A \in \Delta$, meaning that C is defended by Δ.

If B has two or more attackers in $\langle Arg, \rightharpoonup_u \rangle$, we denote the attackers of B by $A_1, \cdots, A_k$. According to the assumption that $\rightharpoonup$ is topological-restricted with respect to t-self-defense, for every pair of attackers A_i and A_j of B, no agent rejects both $A_i \rightharpoonup B$ and $A_j \rightharpoonup B$. We now show that C is defended by Δ in $F(\rightharpoonup)$. If there are two or more arguments in $A_1, \cdots, A_k$ that are included in Δ, we take two of them and denote by A_i and A_j. Clearly, one of $A_i \rightharpoonup B$

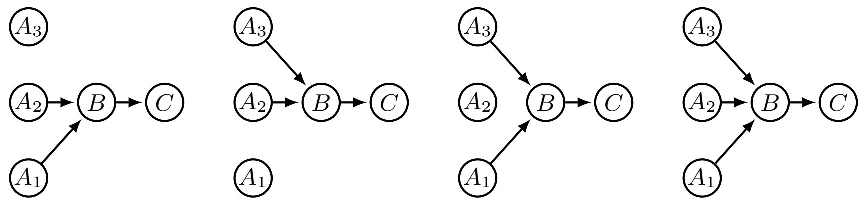

Fig. 3. Scenarios used in Example 3.

and $A_j \rightharpoonup B$ is supported by the majority of agents. Thus, A is defended by Δ in $F(\rightharpoonup)$. If there is only one argument in $A_1, \cdots, A_k$ that is included by Δ, and we denote it by A_i. Clearly, $A_i \rightharpoonup B$ is supported by agents who support $B \rightharpoonup C$, i.e., $A_i \rightharpoonup B$ is accepted by F, meaning that C is defended by Δ in $F(\rightharpoonup)$ as well. For the scenario that no argument in $A_1, \cdots, A_k$ that is included in Δ, we note that this is impossible as for agents who support $B \rightharpoonup C$, Δ is not self-defending in their individual argumentation frameworks.

Let us come back to Example 2, the union of attack-relations of the profile $\rightharpoonup_u = \{A_1 \rightharpoonup B, A_2 \rightharpoonup B, A_3 \rightharpoonup B, B \rightharpoonup C\}$. Clearly, the profile is not topological-restricted with respect to t-self-defense as $B \rightharpoonup C$, whose attacker B has three attackers, and for every pair of attackers of B in $\langle Arg, \rightharpoonup_u \rangle$ there is at least one agent who rejects both.

Example 3. Now we consider the profile illustrated in Figure 3, in which $\rightharpoonup_1 = \{A_1 \rightharpoonup B, A_2 \rightharpoonup B\}, \rightharpoonup_2 = \{A_2 \rightharpoonup B, A_3 \rightharpoonup B\}, \rightharpoonup_3 = \{A_3 \rightharpoonup B, A_1 \rightharpoonup B\}$ and we want to know whether $\{A_1, A_2, A_3, C\}$ is admissible if the outcome of the majority rule. Clearly, the profile is topological-restricted with respect to t-self-defense. We can see that the union of attack-relations of the profile $\rightharpoonup_u = \{A_1 \rightharpoonup B, A_2 \rightharpoonup B, A_3 \rightharpoonup B, B \rightharpoonup C\}$, and for every attack in $B \rightharpoonup C$, for example, for every pair of attackers A_1, A_2, for example, no agent rejects both $A_1 \rightharpoonup B$ and $A_2 \rightharpoonup B$. While $\{A_1, A_2, A_3, C\}$ is admissible in every individual agent's argumentation framework, it is also admissible in the outcome of the majority rule, as expected.

7 Conclusion

In this paper, we have studied the preservation of semantic properties during the aggregation of argumentation frameworks with topological constraints. The topological constraints we consider in this paper include acyclicity, symmetry, as well as t-self-defense, and the semantic properties we consider include conflict-freeness, admissibility, being an extension under a specific semantics, nonemptiness of the grounded extension and coherence. Compared to the preservation results for several semantic properties by Chen and Endriss without constraints showing that only dictatorships preserve them, there are aggregation rules that have some intuitive appeal preserve them with topological constraints. When the restriction under consideration is t-self-defense, we can even preserve admissibility under the majority rule.

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References

1. Arrow, K.J., Sen, A.K., Suzumura, K. (eds.): Handbook of Social Choice and Welfare. North-Holland (2002)
2. Arrow, K.J.: Social Choice and Individual Values. John Wiley and Sons, 2nd edn. (1963), first edition published in 1951
3. Baroni, P., Giacomin, M.: Characterizing defeat graphs where argumentation semantics agree. In: Proc. 1st Int. Workshop on Argumentation and Non-Monotonic Reasoning (ARGNMR07). pp. 33–48 (2007)
4. Baroni, P., Giacomin, M.: On principle-based evaluation of extension-based argumentation semantics. Artificial Intelligence **171**(10-15), 675–700 (2007)
5. Baroni, P., Giacomin, M., Guida, G.: Scc-recursiveness: a general schema for argumentation semantics. Artificial Intelligence **168**(1-2), 162–210 (2005)
6. Baumeister, D., Neugebauer, D., Rothe, J.: Collective acceptability in abstract argumentation. Journal of Applied Logics **2631**(6), 1503 (2021)
7. Bodanza, G.A., Tohmé, F.A., Auday, M.R.: Collective argumentation: A survey of aggregation issues around argumentation frameworks. Argument & Computation **8**(1), 1–34 (2017)
8. Caminada, M., Pigozzi, G.: On judgment aggregation in abstract argumentation. Journal of Autonomous Agents and Multiagent Systems **22**(1), 64–102 (2011)
9. Chen, W.: Collective argumentation: The case of aggregating support-relations of bipolar argumentation frameworks. In: Proceedings of the 18th Conference on Theoretical Aspects of Rationality and Knowledge (TARK). pp. 87–102 (2021)
10. Chen, W.: Guaranteeing admissibility of abstract argumentation frameworks with rationality and feasibility constraints. Journal of Logic and Computation (2021). https://doi.org/10.1093/logcom/exab011, to appear
11. Chen, W., Endriss, U.: Aggregating alternative extensions of abstract argumentation frameworks: Preservation results for quota rules. In: Proceedings of the 7th International Conference on Computational Models of Argument (COMMA). IOS Press (2018)
12. Chen, W., Endriss, U.: Preservation of semantic properties in collective argumentation: The case of aggregating abstract argumentation frameworks. Artificial Intelligence **269**, 27–48 (2019)
13. Coste-Marquis, S., Devred, C., Konieczny, S., Lagasquie-Schiex, M.C., Marquis, P.: On the merging of Dung's argumentation systems. Artificial Intelligence **171**(10–15), 730–753 (2007)
14. Coste-Marquis, S., Devred, C., Marquis, P.: Symmetric argumentation frameworks. In: European Conference on Symbolic and Quantitative Approaches to Reasoning and Uncertainty. pp. 317–328. Springer (2005)

15. Dietrich, F., List, C.: Majority voting on restricted domains. Journal of Economic Theory **145**(2), 512–543 (2010)
16. Dung, P.M.: On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. Artificial Intelligence **77**(2), 321–358 (1995)
17. Dung, P.M., Mancarella, P., Toni, F.: Computing ideal sceptical argumentation. Artificial Intelligence **171**(10), 642–674 (2007)
18. Endriss, U., Grandi, U.: Graph aggregation. Artificial Intelligence **245**, 86–114 (2017)
19. Grossi, D., Pigozzi, G.: Judgment Aggregation: A Primer. Synthesis Lectures on Artificial Intelligence and Machine Learning, Morgan & Claypool Publishers (2014)
20. List, C.: A possibility theorem on aggregation over multiple interconnected propositions. Mathematical Social Sciences **45**(1), 1–13 (2003)
21. Rahwan, I., Tohmé, F.A.: Collective argument evaluation as judgement aggregation. In: Proceedings of the 9th International Conference on Autonomous Agents and Multiagent Systems (AAMAS). pp. 417–424. IFAAMAS (2010)
22. Sen, A.K.: A possibility theorem on majority decisions. Econometrica: Journal of the Econometric Society pp. 491–499 (1966)
23. Tohmé, F.A., Bodanza, G.A., Simari, G.R.: Aggregation of attack relations: A social-choice theoretical analysis of defeasibility criteria. In: Proceedings of the 5th International Symposium on Foundations of Information and Knowledge Systems (FoIKS), pp. 8–23. Springer (2008)