Preservation of Admissibility with Rationality and Feasibility Constraints*

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Abstract. The paper considers the problem of in what circumstances an aggregation rule guarantees an admissible output extension that represents a good compromise between several input extensions of abstract argumentation framework, each provided by a different individual. To achieve this, we introduce the concept of concrete admissibility for abstract argumentations by strengthening Dung's admissibility. We also define a model for extension aggregation that clearly separates the constraint supposed to be satisfied by individuals and the constraint that must be met by the collective decision. Using this model, we show that the majority rule guarantees admissible sets on newly defined admissible sets.

1 Introduction

Admissibility is an importance semantic property of argumentation frameworks. It lies in the heart of all semantics discussed in [8], and is shared by many more recent proposals [2]. Under Dung's argumentation framework [8], a set of arguments satisfies admissibility if it defends all its members in the sense that for any argument A in the set, either A is un-attacked, or if attacked by some argument B, then there is an argument in the set that attacks B, and it does not contain internal attacks.

When a group of agents are confronted with the same abstract argumentation framework, and each of them chooses an extension, we may wish to aggregate such extensions into a collective one, which represents the consensus of the group. Similar question has been received attention in the last decades or so (see, e.g., [17, 5, 1, 4, 6]). In this paper, we address the question of in what circumstances, an aggregation rule will guarantee admissible outputs during aggregation of extensions of abstract argumentation framework. In existing work, we mention Chen and Endriss [6] have shown that no aggregation rule preserves Dungs admissibility in general. Under their settings, all agents report extensions that are admissible, and they aggregate such extensions by making use of a set of

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conceptually and computationally simple aggregation rules, quota rules, which have been studied in depth in judgment aggregation [7].

The graded semantics is a new theory of justification of arguments developed by Grossi and Modgil [13], in which the degree of acceptance of arguments can be weakened or strengthened. In the graded semantics, the number of attackers and defenders are given a fine-grained assignment when deciding whether a specific argument is acceptable. The notion of admissibility is extended to mn-admissibility. Such notion has the potential to require that if a set of arguments Δ is admissible, then any attacker of $A \in \Delta$ is attacked by more than one argument in Δ. While preserving Dung's admissibility is difficult, there is still no good news for the preservation of graded admissibility. Using the model proposed by Chen and Endriss [6], our results show that no quota rule can guarantee admisible outcomes on graded admissble sets. Thus, preserving graded admissibility is difficult as well.

In this paper, we introduce the concept of concrete admissibility and a new model for extension aggregation. When we consider whether an argument A is acceptable with respect to a set of arguments Δ, graded admissibility only considers the number of A's defenders in Δ, while in concrete admissibility, Δ included all defenders of A, i.e., for any attacker B of argument A, Δ includes all attacker of B.

For the model, we point out that in nearly all existing work on extension aggregation, there is only one single type of constraint (see, e.g., [17, 6]). Such constraint is explicitly represented or left implicit. Following the model proposed by Endriss [10] for judgment aggregation [14, 9], we introduce a model for extension aggregation that allows the constraints assumed to be satisfied by the individual agents can be different from the constraints met by the collective decision returned by the aggregation rule. Using this model, we show that the majority rule guarantees admissible outcomes on revised admissible sets.

The paper is organized as follows. In Section 2, we review some of Dung's basic concepts of the theory of abstract argumentation. Section 3 recalls the preservation results of Dung's semantics introduced by Chen and Endriss [6]. In Section 4 we show that preserving new graded semantics yields similar impossibility results. In Section 5, we introduce concrete admissibility and a new model with integrity and feasibility constraints, and illustrates a positive result with majority rule. We conclude in Section 6 outlining some future research directions.

2 Abstract Argumentation

2.1 Abstract Argumentation Framework

In this section, we recall some of the basic concepts of the theory of abstract argumentation first introduced by [8]. An *argumentation framework* is a pair $AF = \langle Arg, \rightharpoonup \rangle$, in which Arg is a finite set of arguments and $\rightharpoonup$ is a binary relation on Arg. We say that A *attacks* B, if $A \rightharpoonup B$ holds for two arguments

$A, B \in Arg$. For $\Delta \subseteq Arg$ and $B \in Arg$, we write $\Delta \rightharpoonup B$ (namely Δ attacks B) in case $A \rightharpoonup B$ for at least one argument $A \in \Delta$. For $\Delta \subseteq Arg$ and $C \in Arg$ we say that Δ *defends* C in case Δ attacks all arguments $B \in Arg$ with $B \rightharpoonup C$. We write 2^{Arg} for the powerset of Arg.

Given an argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$, the question arises which subset Δ of the set of arguments Arg one should accept. Any such set $\Delta \subseteq Arg$ is called an *extension* of AF. Different criteria have been put forward for choosing an extension. While Dung has defined several semantic, notably complete, grounded, preferred, and stable semantics [8], it is worth mentioning that conflict-freeness, being self-defending, and admissibility are fundamental properties supposed to be satisfied by extensions of semantics.

Definition 1. *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework and let $\Delta \subseteq Arg$ be a set of arguments. We adopt the following terminology:*

- *Δ is called conflict-free if there are no arguments $A, B \in \Delta$ such that $A \rightharpoonup B$.*
- *Δ is called self-defending if $\Delta \subseteq \{C \mid \Delta \text{ defends } C\}$.*
- *Δ is called admissible if it is both conflict-free and self-defending.*

Thus, a set of arguments is admissible if it is conflict-free and being self-defending. In the original paper, Dung defines some other semantics, including complete, grounded, preferred, and stable semantics [8]. All of them are are admissibility-based in the sense that every extension of such semantics is admissible.

2.2 Abstract Argumentation Semantics and Propositional Logic

Following the work by Besnard and Doutre [3] and Chen and Endriss [6], we represent the properties of extensions in a purely syntactic manner, using a logical language. So fix an argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$, think of Arg as a set of propositional variables, and let $\mathcal{L}_{AF}$ be the corresponding propositional language. Now extensions $\Delta \subseteq Arg$ correspond to models of formulas in $\mathcal{L}_{AF}$:

- $\Delta \models A$ for $A \in Arg$ if and only if $A \in \Delta$
- $\Delta \models \neg\varphi$ if and only if $\Delta \models \varphi$ is not the case
- $\Delta \models \varphi \wedge \psi$ if and only if both $\Delta \models \varphi$ and $\Delta \models \psi$

Given a formula φ, we use $\text{Mod}(\varphi) = \{\Delta \subseteq Arg \mid \Delta \models \varphi\}$ to denote the set of all models of φ. Every formula φ identifies a property of extensions of AF, namely $\sigma = \text{Mod}(\varphi)$. When using a formula φ to describe such a property of extensions, we usually refer to φ as an *integrity constraint*. The following simple result characterise the properties of being conflict-free and self-defending in terms of integrity constraints expressed in $\mathcal{L}_{AF}$.

Proposition 1. *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework and let $\Delta \subseteq Arg$ be an extension. Then Δ is conflict-free if and only if:*

$$\Delta \models \text{IC}_{CF} \quad \textit{where} \quad \text{IC}_{CF} = \bigwedge_{\substack{A,B \in Arg \\ A \rightharpoonup B}} (\neg A \vee \neg B)$$

That is, Proposition 1 states that $\text{Mod}(\text{IC}_{CF}) = \{\Delta \subseteq Arg \mid \Delta \textit{ is conflict-free}\}$.

Proposition 2. *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework and let $\Delta \subseteq Arg$ be an extension. Then Δ is self-defending if and only if:*

$$\Delta \models \mathrm{IC}_{SD} \quad \textit{where} \quad \mathrm{IC}_{SD} = \bigwedge_{C \in Arg} [C \rightarrow \bigwedge_{\substack{B \in Arg \\ B \rightharpoonup C}} \bigvee_{\substack{A \in Arg \\ A \rightharpoonup B}} A]$$

We can now use the integrity constraints defined above to construct integrity constraints for the property of admissibility:

- Δ is admissible if and only if $\Delta \models \mathrm{IC}_{AD}$ where $\mathrm{IC}_{AD} = \mathrm{IC}_{CF} \wedge \mathrm{IC}_{SD}$.

Example 1. Consider the argumentation framework $AF = \langle \{A, B, C, D\}, \{A \rightharpoonup C, B \rightharpoonup C, C \rightharpoonup D\} \rangle$, as illustrated in Figure 1. Then $\mathrm{IC}_{SD} = (\neg D \vee A \vee B) \wedge (\neg C)$, $\mathrm{IC}_{AD} = (\neg D \vee A \vee B) \wedge (\neg C) \wedge (\neg A \vee \neg C) \wedge (\neg B \vee \neg C) \wedge (\neg C \vee \neg D)$.

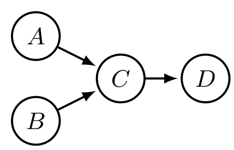


Fig. 1: An AF with four arguments

3 Preservation of the Dung's Admissibility

3.1 Extension Aggregation

In this section, we recall a model for extension aggregation defined by Chen and Endriss [6]. Such model allows a single type of constraint. Fix an argumentation framework $AF = \langle Arg, \rightharpoonup \rangle$. Let $U = \{1, \ldots, u\}$ be a finite set of *agents*. Suppose each agent $i \in U$ supplies us with an extension $\Delta_i \subseteq Arg$, reflecting her individual views of what constitutes an acceptable set of arguments in the context of AF. Thus, we are supplied with a *profile* $\boldsymbol{\Delta} = (\Delta_1, \ldots, \Delta_u)$, a vector of extensions, one for each agent. An *aggregation rule* is a function $F : (2^{Arg})^u \rightarrow 2^{Arg}$, mapping any given profile of extensions to a single extension.

Definition 2. *A quota rule F_q with quota q is the aggregation rule mapping any given profile of extensions to the extension including exactly those arguments accepted by at least q agents.*

The quota rules have low computational complexity in the sense that it is straightforward to compute outputs [11]. The nomination rule is the quota rule with quota $q = 1$. The majority rule is another example of quota rules for which its quota $q = \lceil \frac{u+1}{2} \rceil$.

3.2 Preservation of Dung's Admissibility

Chen and Endriss [6] have considered the problem of aggregation of alternative extensions by making use of quota rules. They exploit known encodings of argumentation semantics in propositional logic. They study the preservation of semantic properties of extensions, including conflict-freeness, self-defending, reinstating, admissibility, and I-Maximal properties. Our focus is admissibility.

Proposition 3. *(Chen and Endriss, 2018) Every quota rule F_q for u agents with a quota $q > \frac{u}{2}$ preserves admissibility for all argumentation frameworks AF with* $\mathrm{MaxDef}(AF) \leqslant 1$.

Note that $\mathrm{MaxDef}(AF)$ is the maximum number of attackers of an argument that itself is the source of an attack.

Theorem 1. *(Chen and Endriss, 2018) No quota rule preserves admissibility for all argumentation frameworks.*

Thus, no quota rule can guarantee the preservation of admissibility in general.

4 Preservation of Graded Admissibility

4.1 Graded Semantics

In this part, we present the graded semantics introduced by Grossi and Modgil [13]. The graded semantics can be seen as a generalisation of Dungs semantics. Extensions of the graded semantics are weakened or strengthened depending on level of self-defending and conflict-freeness they meet.

An argument A is defended by a set of arguments Δ whenever A is attacked by some argument B, there at least one argument in Δ that attacks B. Grossi and Modgil generalize the notion of defense to obtain the notion of *graded defense* [13].

Definition 3. *The defense function is defined as follows. For any $\Delta \subseteq Arg$:*

$$d(\Delta) = \{X \in Arg \mid \forall Y \in Arg : \text{ IF } Y \rightharpoonup X \text{ THEN } \Delta \rightharpoonup Y\} \tag{1}$$

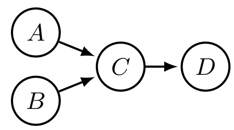
Definition 4. *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework, and m and n be two positive integers $(m, n > 0)$. The graded defense function for Δ is defined as follows. For any $\Delta \subseteq Arg$:*

$$d_n^m(\Delta) = \left\{X \in Arg \text{ s.t. } |\{Y \in \overline{X} \text{ s.t. } |\overline{Y} \cap \Delta| < n\}| < m\right\} \tag{2}$$

where $\bar{X}$ denotes $\{Y \in Arg \mid Y \rightharpoonup X\}$.

So, $d_n^m(\Delta)$ denotes the set of arguments that have at most $m-1$ attackers that are not counter-attacked by at least n arguments in Δ.

Example 2. Let us consider the argumentation framework depicted below.



Let $\Delta = \{A, D\}$, it is easy to verify that $D \in d_1^1(\Delta)$ but $D \notin d_2^1(\Delta)$.

Definition 5. *A set of arguments Δ is said to be acceptable at grade mn(or, mn-acceptable) whenever all of its arguments are such that at most $m-1$ of their attackers are not counter-attacked by at least n arguments in Δ.*

Definition 6. *A set of arguments Δ is said to be mn-self-defending whenever all of its arguments are such that at most $m-1$ of their attackers are not counter-attacked by at least n arguments in Δ.*

Definition 7. *A set of arguments Δ is said to be at grade mn-admissible whenever Δ is mn-acceptable and being conflict-free.*

In fact, when $m = n = 1$, we recover the standard definition of being self-defending, admissibility. It is worth mentioning that Grossi and Modgil define graded admissibility as mn-acceptability plus l-conflict-freeness (a set of arguments Δ is said to be l-conflict-free whenever no arguments $A \in \Delta$ such that A is attacked by at least l arguments in Δ [13]). But for the sake of simplicity, we define graded admissibility as mn-acceptability plus Dung's notion of conflict-freeness.

4.2 Preservation Result For Graded Admissibility

In this section, we start with encoding the property of being graded self-defending in propositional logic and show a preservation result for such property. We then present a result for the property of graded admissibility. The following simple result characterises the property of being graded self-defending in terms of the integrity constraint expressed in $\mathcal{L}_{AF}$.

Proposition 4. *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework and let $\Delta \subseteq Arg$ be an extension. Then Δ is mn-self-defending if and only if:*

$$\Delta \models \mathrm{IC}_{mnSD} \quad \textit{where} \quad \mathrm{IC}_{mnSD} = \bigwedge_{C \in Arg} [C \rightarrow \bigvee_{\{B_1,\dots,B_{(|\bar{C}|-m+1)}\} \in \binom{|\bar{C}|}{|\bar{C}|-m+1}} (\bigwedge_{i=1}^{|\bar{C}|-m+1} (\bigvee_{\{A_1,\dots,A_n\} \in \binom{|B_i|}{n}} (\bigwedge_{j=1}^{n} A_j)))] \tag{3}$$

To get the preservation results for being graded self-defending, we need a result regarding binary aggregation with integrity constraints [12], a variant of judgment aggregation.

Lemma 1. *(Grandi and Endriss, 2013) Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework and let φ be a clause in $\mathcal{L}_{AF}$ with k_1 positive literals and k_2 negative literals. Then a quota rule F_q for u agents preserves the property* $\mathrm{Mod}(\varphi)$ *if and only if:*

$$q \cdot (k_2 - k_1) > u \cdot (k_2 - 1) - k_1 \tag{4}$$

Note that a clause is a disjunction of literals, all integrity constraints can be translated into conjunctions of clauses. The following result shows that if we know the preservation result for some clauses, then we know results for the conjunction of such clauses.

Lemma 2. *(Grandi and Endriss, 2013) Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework, let φ_1 and φ_2 be integrity constraints in $\mathcal{L}_{AF}$, and let F be an aggregation rule that preserves both* $\mathrm{Mod}(\varphi_1)$ *and* $\mathrm{Mod}(\varphi_2)$*. Then F also preserves* $\mathrm{Mod}(\varphi_1 \wedge \varphi_2)$.

Thus, given a quota rule F_q and some clauses $\varphi_1, \dots \varphi_l$, if F_q satisfies all clause φ_i, then it preserves $\mathrm{Mod}(\varphi_1 \wedge \cdots \wedge \varphi_l)$.

Example 3. Given an integrity constraint $\varphi = (\neg A \vee \neg B) \wedge C$. By Lemma 1, a quota rule preserves $\neg A \vee \neg B$ only if $q \cdot (2-0) > u \cdot (2-1) - 0$, i.e., only if $q > \frac{u}{2}$. A quota rule preserves C only if $q \cdot (0-1) > u \cdot (0-1) - 1$, which is always the case, thus, C is preserved by every quota rule. Thus, by Lemma 2, a quota rule preserves φ only if $q > \frac{u}{2}$.

Recall that the nomination rule is the quota rule for which its quota is 1.

Proposition 5. *The nomination rule preserves the property of being a mn-self-defending set.*

Proof. Recall that IC_{mnSD} is a conjunction of formulas of the form

$$C \;\rightarrow \bigvee_{\{B_1,\dots,B_{(|\bar{C}|-m+1)}\} \in \binom{|\bar{C}|}{|\bar{C}|-m+1}} \Big(\bigwedge_{i=1}^{|\bar{C}|-m+1} \Big(\bigvee_{\{A_1,\dots,A_n\} \in \binom{|B_i|}{n}} \Big(\bigwedge_{j=1}^{n} A_j \Big)\Big)\Big)$$

which can be rewritten as

$$C \;\rightarrow \bigwedge_{B_1,\dots,B_m \in \binom{|\bar{C}|}{m}} [\bigvee_{i=1}^{c_1} (A_{\pi_i(1)} \wedge \cdots \wedge A_{\pi_i(n)})_1] \vee \cdots \vee [\bigvee_{i=1}^{c_m} (A_{\pi_i(1)} \wedge \cdots \wedge A_{\pi_i(n)})_m], \tag{5}$$

where $c_i = \binom{|\bar{B}_i|}{n}$ for $i = 1, \cdots, m$, respectively. We take one such clause

$$C \;\rightarrow [\bigvee_{i=1}^{c_1} (A_{\pi_i(1)} \wedge \cdots \wedge A_{\pi_i(n)})] \vee \cdots \vee [\bigvee_{i=1}^{c_m} (A_{\pi_i(1)} \wedge \cdots \wedge A_{\pi_i(n)})], \tag{6}$$

which can be rewritten as

$$C \rightarrow \bigwedge_{j=1}^{n}[\bigvee_{i=1}^{m}(A_{\pi_i(j)} \vee \cdots \vee A_{\pi_{(|\bar{B}_j|-n)}(j)})]. \tag{7}$$

We take one such clause

$$C \rightarrow [\bigvee_{i=1}^{m}(A_{\pi_i(j)} \vee \cdots \vee A_{\pi_{(|\bar{B}_j|-n)}(j)})]. \tag{8}$$

Its number of positive literals is $(|\bar{B}_j| - n) \cdot m$, its number of negative literals is 1. Thus, according to Lemma 1, a uniform quota rule with quota q preserves it if and only if $q < \frac{(|\bar{B}_j|-n)\cdot m}{(|\bar{B}_j|-n)\cdot m-1}$. As IC_{mnSD} is a conjunction of such clauses, therefore we need to satisfy this inequality for all relevant m, n and B_j. This requirement is most demanding for largest values of n, and smallest of m and B_j. However, we point out that if $q = 1$, then $q < \frac{(|\bar{B}_j|-n)\cdot m}{(|\bar{B}_j|-n)\cdot m-1}$ is always the case. Thus, we have the proposition.

Theorem 2. *No quota rule preserves mn-admissibility for all argumentation frameworks.*

Proof. Recall that standard definition of admissibility is a special case of mn-admissibility for which $m = n = 1$. By Theorem 1, we get that no quota rule preserves 11-admissibility. Thus, we have the theorem.

Thus, we obtain a similar result for mn-admissibility.

5 Preservation Results For Concrete Admissbility

5.1 Concrete Admissibility

The graded semantics provides a theory of degree of justification of arguments. Under the graded semantics, the assignment of status of arguments are defined by the numbers of attackers and defenders. Theses graded semantics provide ways of strengthening or weakening the standard Dung semantics. While grade semantics appeals to the numbers of attackers and defenders to define acceptability of arguments, it is worth mentioning that, in some scenarios, given two arguments for which the numbers of attackers and defenders of such pair of arguments are different, but they share similar features. Consider the following example.

Example 4. Let us consider two sets of arguments $\Delta_1 = \{C, A\}$ in AF_1, $\Delta_2 = \{C, A, D, E\}$ in AF_2, as illustrated in Figure 2. The numbers of defenders of C in AF_1 and AF_2 are different: C has one defender in AF_1, and C has three defenders in AF_2. But both of them are concretely defended in the sense that

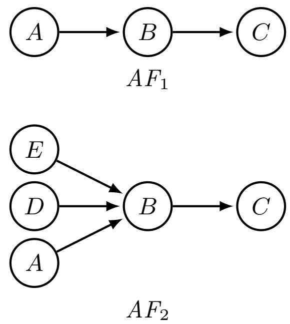


Fig. 2: Two argumentation frameworks

for every argument A, if C is defended by A, then A is included in Δ. Argument C has one defender in AF_1, namely A, and A is included in Δ_1; Argument C have three defenders in AF_2, namely A, D, E, and they are included in Δ_2. The similarity of C between AF_1 and AF_2 is not captured by Modgil and Grossi's graded semantics.

In above example, arguments share the same degree of justification of acceptability of arguments, can have different numbers of defenders.

Take an argument $A \in Arg$ and a set of arguments $\Delta \subseteq Arg$, under Dung's admissibility, we say that Δ defends A if for every attack $B \in Arg$ of argument A, Δ accepts at least one attacker of B, we say that Δ is admissible if Δ defends all its members and being conflict-free. We introduce the concept of *concrete admissibility*. We say that Δ *concretely defends* A if for every attacker B of arguments A, Δ accepts *all* attackers of B, i.e, Δ includes all defenders of A, we say that Δ satisfies concrete admissibility if Δ concretely defends all its members and Δ is conflict-free. Note that the requirement of concrete acceptability of arguments is a strong requirement.

We use the notion of concrete defense to define concrete admissibility.

Definition 8. *Take an argument $A \in Arg$ and a set of arguments $\Delta \subseteq Arg$. We say that Δ* concretely defends *A if Δ for every attacker B of arguments A, Δ accepts* all *attackers of B.*

For example, in Figure 2, $\{A\}$ concretely defends C in AF_1, $\{A, D, E\}$ concretely defends C in AF_2.

Definition 9. *Take an argument $A \in Arg$ and a set of arguments $\Delta \subseteq Arg$. We say that Δ is* concretely admissible *if Δ concretely defends all of its members, and Δ is conflict-free.*

Recall that we use φ to refer a property of extensions, or an integrity constraint. The following simple result characterises the properties of being concrete defending in terms of the integrity constraint expressed in $\mathcal{L}_{AF}$.

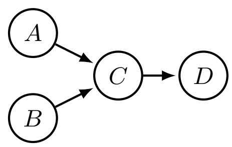
Proposition 6. *Let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework and let $\Delta \subseteq Arg$ be an extension. Then Δ is concrete defending if and only if:*

$$\Delta \models \mathrm{IC}_{CD} \quad \textit{where} \quad \mathrm{IC}_{CD} = \bigwedge_{C \in Arg} \bigwedge_{\substack{B \in Arg \\ B \rightharpoonup C}} \bigwedge_{\substack{A \in Arg \\ A \rightharpoonup B}} [C \to A]$$

We can now use the integrity constraint defined above to construct the integrity constraint for the the property of concrete admissibility:

– Δ is concretely admissible if and only if $\Delta \models \mathrm{IC}_{CA}$ where $\mathrm{IC}_{CA} = \mathrm{IC}_{CF} \wedge \mathrm{IC}_{CD}$.

Example 5. Consider the argumentation framework $AF = \langle \{A, B, C, D\}, \{A \rightharpoonup C, B \rightharpoonup C, C \rightharpoonup D\} \rangle$. Then $\mathrm{IC}_{SD} = (\neg D \vee A \vee B) \wedge (\neg C)$, $\mathrm{IC}_{CD} = (\neg D \vee A) \wedge (\neg D \vee B) \wedge (\neg C)$. In this example, $\{A, D\}$ and $\{B, D\}$ are admissible, but they are not concretely admissible. $\{A, B, D\}, \{A\}, \{B\}, \{A, B\}, \emptyset$ are all admissible and concretely admissible sets.



5.2 Concrete Admissibility and Prime Implicates

In the section, we generalize the notion of *prime implicate* to our context, a clause $\pi \in \mathcal{L}_{AF}$ is a prime implicate of a formula $\Gamma \in \mathcal{L}_{AF}$ if (i) $\Gamma \models \pi$ and (ii) for every clause $\pi' \in \mathcal{L}_{AF}$ with $\Gamma \models \pi'$ if $\pi' \models \pi$ then $\pi = \pi'$ [16]. In other words, the prime implicates are the logically strongest clauses entailed by Γ.

Recall that a clause is a disjunction of literals. A clause is simple if it has at most two literals, a clause is nonsimple if it cannot be simplified to a clause with less than three literals. A formula is simple if it logically equivalent to a conjunction of clauses with at most two literals (it is also called Krom formula). We first present three results concerning prime implicates.

Fact 3 *A formula Γ is simple if and only if all its prime implicates are simple.*

Lemma 3. *(Marquis, 2000) If $\Gamma \models \Gamma'$ is the case, then for every prime implicate π' of Γ' there exists a prime implicate π of Γ such that $\pi \models \pi'$.*

Definition 10. *(Endriss, 2018) A pair of formulas (Γ, Γ') is simple, if for every nonsimple prime implicate π' of Γ' there exists a simple prime implicate π of Γ such that $\pi \models \pi'$.*

Using the results above, we are now ready to present some results concerning the relation between self-defending and concrete defending, and the relation between Dung's admissibility and concrete admissibility.

Lemma 4. $\mathrm{IC}_{CD} \models \mathrm{IC}_{SD}$.

Proof. Recall that IC_{SD} is a conjunction of a collection of formulas of the form $C \to \bigwedge_{\substack{B\in Arg\\ B\rightharpoonup C}} \bigvee_{\substack{A\in Arg\\ A\rightharpoonup B}} A$. We take the one indexed by $C \in Arg$ and rewrite as $\bigwedge_{\substack{B\in Arg\\ B\rightharpoonup C}} (\neg C \vee \bigvee_{\substack{A\in Arg\\ A\rightharpoonup B}} A)$. This formula is a conjunction of a collection of clauses of the form $(\neg C \vee \bigvee_{\substack{A\in Arg\\ A\rightharpoonup B}} A)$. We take the one indexed by $B \in Arg$ with $B \rightharpoonup C$ and rewrite as $(\neg C \vee A_1 \vee A_2 \vee \ldots \vee A_n)$, in which $A_1, A_2, \ldots A_n$ defend C by attacking B. We denote it by φ'. Obviously φ' is a clause of IC_{SD}. We are going to show that there is at least one clause φ of IC_{CD} such that $\varphi \models \varphi'$.

Recall that IC_{CD} is a conjunction of a collection of formulas of the form $\bigwedge_{\substack{B\in Arg\\ B\rightharpoonup C}} \bigwedge_{\substack{A\in Arg\\ A\rightharpoonup B}} [C \to A]$. Let us consider one such formulas $\bigwedge_{\substack{B\in Arg\\ B\rightharpoonup C}} \bigwedge_{\substack{A\in Arg\\ A\rightharpoonup B}} [C \to A]$ which indexed by $C \in Arg$. This formula is a conjunction of a collection of formulas indexed by an argument $B \in Arg$ with $B \rightharpoonup C$. Let us consider one formula $\bigwedge_{\substack{A\in Arg\\ A\rightharpoonup B}} [C \to A]$ which indexed by $B \in Arg$ with $B \rightharpoonup C$. This formula can be rewritten as $(\neg C \vee A_1) \wedge (\neg C \vee A_2) \wedge \ldots \wedge (\neg C \vee A_n)$ in which $A_1, A_2, \ldots A_n$ defend C by attacking B as well. We denote it by $\varphi_1 \wedge \varphi_2 \wedge \ldots \wedge \varphi_n$. Since $A_1, A_2, \ldots A_n$ defending C by attacking B, we know that $(\neg C \vee A_i) \models (\neg C \vee A_1 \vee A_2 \vee \ldots \vee A_n)$ for $i \in \{1, 2, \ldots, n\}$. Thus, $\varphi_i \models \varphi'$ for $i \in \{1, 2, \ldots, n\}$.

Using the same construction, we can show that for every clause φ' of IC_{SD}, there is at least one clause φ of IC_{CD} such that $\varphi \models \varphi'$. Thus, $\mathrm{IC}_{CD} \models \mathrm{IC}_{SD}$.

Proposition 7. $(\mathrm{IC}_{CD}, \mathrm{IC}_{SD})$ *is simple.*

Proof. From Lemma 4, we know that $\mathrm{IC}_{CD} \models \mathrm{IC}_{SD}$. With Lemma 3, we know that for every prime implicate π' of IC_{SD} there exists a prime implicate π of IC_{CD} such that $\pi \models \pi'$. Obviously IC_{CD} is a conjunction of clauses with at most two literals. Thus, it is simple. By Fact 3, we have that every prime implicate of IC_{CD} is simple.

Putting together the above facts we are able to conclude that for every (simple and nonsimple) prime implicate π' of IC_{SD} there exists a simple prime implicate π of IC_{CD} such that $\pi \models \pi'$, and we are done.

Lemma 5. $\mathrm{IC}_{CA} \models \mathrm{IC}_{AD}$.

Proof. Recall that $\mathrm{IC}_{CA} = \mathrm{IC}_{CD} \wedge \mathrm{IC}_{CF}$, $\mathrm{IC}_{AD} = \mathrm{IC}_{SD} \wedge \mathrm{IC}_{CF}$. By Lemma 4, we get that $\mathrm{IC}_{CD} \models \mathrm{IC}_{SD}$. Thus, we have $\mathrm{IC}_{CA} \models \mathrm{IC}_{AD}$.

Proposition 8. $(\mathrm{IC}_{CA}, \mathrm{IC}_{AD})$ *is simple.*

Proof. Putting Lemma 5 and Lemma 3 together we get that for every prime implicate π' of IC_{AD} there exists a prime implicate π of IC_{CA} such that $\pi \models \pi'$. Since IC_{CF} is a conjunction of clauses with at most two literals, we know that IC_{CA} is a conjunction of clauses with at most two literals as well, i.e., IC_{CA} is simple. Thus, with Fact 3 we get that every (simple and nonsimple) prime implicate π' of IC_{AD} there exists a simple prime implicate π of IC_{CA} such that $\pi \models \pi'$. We are done.

5.3 A Model with Rationality and Feasibility Constraints

In nearly all existing work on judgment aggregation [12, 15] as well as some work on extension aggregation [6], only a single type of constraint, namely the *integrity constraint* is considered. Integrity constraints decide what is permissible for both the input and the output. As we have shown in Section 3 and Section 4, Dung's admissibility and graded admissibility fail to be preserved under the model that allows for integrity constraints only. In this section, we propose a new model that allows for a distinct between rationality constraints and feasibility constraints. Let us illustrate the model with an example, adapted from [10]:

Example 6. A university council with 5 members needs to decide on the funding for three projects: (φ_1): refurbishing the university stadium, (φ_2): organising an international conference, (φ_3): building a new student dormitory. The budget is limited and it is not feasible to fund all three projects. However, the councilors are not required to keep this constraint in mind when casting their votes on the projects. Instead, they assumed to please at least one of the issues, i.e., it would be irrational for a councilor not to recommend any of the projects for funding. Suppose their votes are as follows:

	φ_1	φ_2	φ_3
Councillor 1	1	1	0
Councillor 2	0	0	1
Councillor 3	1	0	1
Councillor 4	1	1	0
Councillor 5	1	1	1

Table 1: Scenario used in Example 6

Thus, every council's vote meets the rationality constraint. However, the outcome of the majority rule violates the feasibility constraint.

We reuse terminologies introduced in Section 3: let $AF = \langle Arg, \rightharpoonup \rangle$ be an argumentation framework, let U be a finite set of agents. Suppose that every agent provides an extension Δ_i, which gives rise to a profile of extensions $\boldsymbol{\Delta} = \{\Delta_1, \ldots, \Delta_u\}$. A profile is Γ-rational if $\Delta_i \models \Gamma$ for all $i \in U$. Thus, we use Γ to define the permissible profiles of extensions, which is called a *rationality constraint.* An outcome is Γ'-feasible if the outcome satisfies such constraint. We call Γ' a *feasibility constraint*, which defines the acceptable outcomes.

Definition 11. *An aggregation rule $F : (2^{Arg})^u \to 2^{Arg}$ is said to guarantee Γ'-feasible on Γ-rational profiles if for every profile $\boldsymbol{\Delta} \in \mathrm{Mod}(\Gamma)^u$ it is the case that $F(\boldsymbol{\Delta}) \in \mathrm{Mod}(\Gamma')$.*

Thus, we say F guarantees Γ'-feasible outcomes on Γ-rational profiles if for any profile $\boldsymbol{\Delta}$ for which $\Delta_i \models \Gamma$ for all $i \in U$ is the case, we have $\Delta \models \Gamma'$.

5.4 Preservation Results For Concrete Admissibility

In this section, we are ready to present a positive result for obtaining a admissible set on concretely admissible sets with rationality and feasibility constraints. Before going any further, we show a result from Endriss [10], which is needed to prove our main result.

Theorem 4. *(Endriss, 2018) The majority rule guarantees Γ'-feasible outcomes on Γ-rational profiles if and only if $\Gamma \models \Gamma'$ and (Γ, Γ') is simple.*

Theorem 5. *The majority rule guarantees* IC_{SD}*-feasible outcomes on* IC_{CD}*-rational profiles.*

Proof. This therom is a consequence of Lemma 4, Proposition 7, and Theorem 4.

In [6], we have shown that no uniform quota rule preserves admissibility for all argumentation frameworks. In contrast to this, we have a relatively positive result when the profiles we are considering are strengthened to concrete admissibility.

Theorem 6. *The majority rule guarantees admissible outcomes on concretely admissible profiles.*

Proof. This theorem is a consequence of Lemma 5, Proposition 8, and Theorem 4.

6 Conclusion

In this paper, we have explored the possibility of obtaining an admissible set of arguments during the aggregation of extensions of an abstract argumentation framework. We have introduced the concrete admissibility, which allows for strong assignments of status to arguments. To achieve this, we have proposed a model that allows for a clear distinction between integrity and feasible constraints, which is supposed to be satisfied by individual decisions and collective decisions, respectively. We have shown the majority rule, a fair rule that is appealing on normative grounds, guarantees admissible sets on concrete admissible sets. In this paper, only admissibility is considered. Even though admissibility is a fundamental property of extension of argumentation framework, other properties are of particular interest as well. Thus, it is interesting to formulate variants for other semantics based on concrete admissibility, such as completeness, preferredness, stability, and consider the preservation of such semantic properties by making use of our new model. It would be natural to investigate whether it is possible to obtain positive results for such semantic properties using our new model.

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