Aggregating Alternative Extensions of AAFs:

Preservation Results for Quota Rules

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Objectives and Outline

Motivatd by the challenge of modelling collective argumentation, we consider the problem of *aggregating* extensions of AAFs and study the preservation results for quota rules.

We make use of results from two fields:

- argumentation integrity constraints for semantics
- binary aggregation with integrity constraints

I will present:

- the problem of preservation when aggregating extensions
- examples for preservation results, sketching some of our techniques

Aggregation of Extensions

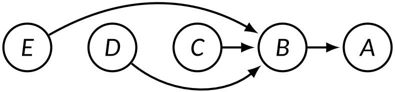
Fix an $AF = \langle Arg, \rightharpoonup \rangle$. Suppose each agent supplies us with an *extension* reflecting her *individual views* of what constitutes an acceptable set of arguments in the context of AF. We would like to aggregate this information by making use of *quota rules*.

Terminology: The quota rule F_q with quota q is defined as $F_q(\mathbf{\Delta}) = \{A \in Arg \mid \#\{i \in N \mid A \in \Delta_i\} \geqslant q\}$ where n is the number of agents and $\mathbf{\Delta} = (\Delta_1, \ldots, \Delta_n)$ is a profile of extensions.

Related work: Rahwan and Tohmé, Caminada and Pigozzi

An Example

Suppose three agents evaluate the following AF:



They report the extensions $\{A, C\}$, $\{A, D\}$, and $\{A, E\}$, respectively, all of which are admissible. But applying the majority rule (i.e., the quota rule F_q with $q = \lceil \frac{n}{2} \rceil$) yields $\{A\}$, which is *not* admissible!

Research Question: Which properties are preserved by which quota rules?

Integrity Constraints for Semantics

Let $AF = \langle Arg, \rightharpoonup \rangle$ be an AF and let $\Delta \subseteq Arg$ be an extension. Then Δ is conflict-free (self-defending, reinstating) iff:

$$\Delta \models \mathrm{IC}_{CF} \quad \text{where} \quad \mathrm{IC}_{CF} = \bigwedge_{\substack{A,B \in Arg \\ A \rightharpoonup B}} (\neg A \vee \neg B)$$

Δ is self-defending iff:

$$\Delta \models \mathrm{IC}_{SD} \quad \text{where} \quad \mathrm{IC}_{SD} = \bigwedge_{C \in Arg} [C \rightarrow \bigwedge_{\substack{B \in Arg \\ B \rightharpoonup C}} \bigvee_{\substack{A \in Arg \\ A \rightharpoonup B}} A]$$

Δ is admissible iff $\Delta \models \mathrm{IC}_{CF} \wedge \mathrm{IC}_{SD}$,

Terminology: Δ is *self-defending* if $\Delta \subseteq \{C \mid \Delta \text{ defends } C\}$.

P. Besnard and S. Doutre. Checking the acceptability of a set of arguments. In *Proc. of N.M.R*, 2004.

Extension Aggregation with Integrity Constraints

Given an *intrgrity constraint* $\varphi = p_1 \vee, \ldots, \vee p_n$ with k_1 *positive* literals and k_2 *negative* literals, a quota rule F_q with quota $q \in \{1, \ldots, n\}$ preserves the property $\mathrm{Mod}(\varphi)$ if and only if:

$$q \cdot (k_2 - k_1) \quad > \quad n \cdot (k_2 - 1) - k_1$$

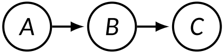
If F preserves both $\mathrm{Mod}(\varphi_1)$ and $\mathrm{Mod}(\varphi_2)$. Then F also preserves $\mathrm{Mod}(\varphi_1 \wedge \varphi_2)$ (Grandi and Endriss, 2013).

Example: the quota rule with $q > \frac{n}{2}$ preserves $\neg A \vee \neg B$, the quota rule with $q > \frac{2 \cdot n}{3}$ preserves $\neg C \vee \neg D \vee \neg E$, then the quota rule with $q > \frac{2 \cdot n}{3}$ preserves $(\neg A \vee \neg B) \wedge (\neg C \vee \neg D \vee \neg E)$

Preserving Conflict-Freeness

A quota rule F_q for n agents preserves *conflict-freeness* for AF if and only if $q > \frac{n}{2}$:

- The integrity constraint for CF is $\bigwedge_{\substack{A,B \in Arg \\ A \rightharpoonup B}} (\neg A \vee \neg B)$ (Besnard and Doutre, 2004)
- For any quota $q > \frac{n}{2}$, F_q preserves the clauses of the form $\neg A \vee \neg B$
- Thus, F_q preserves the conjunction of clauses of the form $\neg A \vee \neg B$, namely preserves IC_{CF}

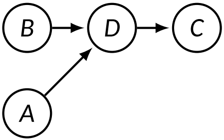


The IC for CF for the above AF is $(\neg A \vee \neg B) \wedge (\neg B \vee \neg C)$.

Preserving Self-defense

A quota rule F_q for n agents preserves *self-defense* for an *AF* if $q \cdot (\mathrm{MaxDef}(AF) - 1) < \mathrm{MaxDef}(AF)$:

- The integrity constraint for SD is $\bigwedge_{C \in Arg} [\, C \rightarrow \bigwedge_{\substack{B \in Arg \\ B \rightharpoonup C}} \bigvee_{\substack{A \in Arg \\ A \rightharpoonup B}} A \,]$
- $C \rightarrow \bigwedge_{\substack{B \in Arg \\ B \rightharpoonup C}} \bigvee_{\substack{A \in Arg \\ A \rightharpoonup B}} A$ can be rewrite as $\bigwedge_{\substack{B \in Arg \\ B \rightharpoonup C}} (\neg C \vee \bigvee_{\substack{A \in Arg \\ A \rightharpoonup B}} A)$, and F preserves it iff $q \cdot (k_{C,B} - 1) < k_{C,B}$
- the *largest* value of $k_{C,B}$ is $\mathrm{MaxDef}(AF)$
- we satisfy all inequalities in case $q \cdot (\mathrm{MaxDef}(AF) - 1) < \mathrm{MaxDef}(AF)$



The IC for SD for the above AF is $D \rightarrow (A \vee B)$, rewritten as $\neg D \vee A \vee B$.

Terminology: $\mathrm{MaxDef}(AF)$ denotes the *maximum number* of attackers of an argument that itself is the source of an attack.

Preserving Admissibility

- The *nomination rule* preserves the property of *self-defense* for all argumentation frameworks.
- Every quota rule F_q for n agents with a quota $q > \frac{n}{2}$ preserves *admissibility* for all argumentation frameworks AF with $\mathrm{MaxDef}(AF) \leqslant 1$.
- *No quota rule* preserves *admissibility* for all argumentation frameworks.

Preservation Results

Property	Constraint(s)	Uniform Quota Rule(s)
Conflict-freeness		$q > \frac{n}{2}$
Self-defending		$q \cdot (\mathrm{MaxDef}(AF) - 1) < \mathrm{MaxDef}(AF)$
Self-defending		*Nomination rule*
Admissibility	$\mathrm{MaxDef}(AF) \leqslant 1$	$q > \frac{n}{2}$
Admissibility		*None*
Being Reinstating		$q \cdot (\mathrm{MaxAtt}(AF) - 1) > n \cdot (\mathrm{MaxAtt}(AF) - 1) - 1$
Being Reinstating		*Unanimity rule*
Completeness	$\mathrm{MaxDef}(AF) \leqslant 1$	$q > \frac{n}{2}$
Completeness		*None*
I-Maximal property σ	$\mid \sigma \mid \geqslant 2$ and n is even	*None*
I-Maximal property σ	$\mid \sigma \mid \geqslant 2$ and n is odd	*No quota rule different from the majority rule*
Property σ	$\mid \sigma \mid = 2$	*Majority rule*

Summary and Furture Works

We have seen:

- encoding of argumentation semantics in *propositional logic* along with prior work in *judgment aggregation* establish positive results in extension aggregation.
- *social choice theory* can be fruitfully applied to the analysis of scenarios of *collective argumentation*.

Future work: further properties of extensions, other aggregation rules besides the quota rules, other types of argumentation formalisms, . . .